Strategy sequential difficulty effects vary with working-memory and response–stimulus-intervals: A study in arithmetic

Kim Uittenhove, Patrick Lemaire *

Laboratoire de Psychologie Cognitive, Centre National de la Recherche Scientifique and Aix-Marseille Université, Marseille, France

Abstract

Strategy sequential difficulty effects are the findings that when participants execute strategies, performance is worse after a difficult strategy than after an easy strategy (Uittenhove & Lemaire, 2012). Strategy sequential difficulty effects are hypothesized to result from decreased working-memory resources following difficult strategy execution. In the present study we found a correlation between individuals’ working memory and strategy sequential difficulty effects in arithmetic, supporting a working-memory account of these effects. Furthermore, we varied response–stimulus intervals, and we found decreased strategy sequential difficulty effects with increasing response–stimulus intervals. Implications of these findings for further understanding of strategic variations in human cognition are discussed.

1. Introduction

Sequential difficulty effects in arithmetic have first been discovered by Schneider and Anderson (2010). In three experiments, participants had to switch between tasks (addition and subtraction) in an arithmetic problem verification task. The authors manipulated the difficulty of problems (problems with or without carry-over in Expt. 1; vertical or horizontal format in Expt. 2; and true or false problems in Expt. 3). They found that participants took more time to solve a problem if this problem followed a hard problem than if it followed an easy problem.

Uittenhove and Lemaire (2012) took a strategy approach to problem solving and proposed that problem sequential difficulty effects could actually be associated to the different strategies used on different problems. For example, in Schneider and Anderson’s Experiment 2, vertical presentation of arithmetic problems could have elicited a columnar-retrieval strategy whereas horizontal presentation could have elicited a more demanding procedural strategy (Geary, Frensch, & Wiley, 1993; Green, Lemaire, & Dufau, 2007). The difficulty of strategies elicited by problems could thus have led to sequential difficulty effects in Schneider and Anderson (2010).

To test the possibility of strategy sequential difficulty effects, Uittenhove and Lemaire (2012) asked participants to accomplish a computational estimation task (e.g., estimating the solution to two-digit addition problems) with imposed rounding strategies. Rounding strategies can differ in difficulty. For example, the rounding-up strategy (e.g., doing 50 + 70 to estimate 47 + 68) has been found to be more difficult than the rounding-down strategy (e.g., doing 40 + 60 to estimate 43 + 62) (e.g., LeFevre, Greenham, & Waheed, 1993; Lemaire, Arnaud, & Lecacheur, 2004; Lemaire & Lecacheur, 2010). The rounding-down strategy is easiest because it does not require the extra step of incrementing operands and keeping them in WM. The rounding-up strategy is more difficult, because it requires incrementing and maintaining two operands in WM. Uittenhove and Lemaire (2012) found that participants were faster following the easier rounding-down strategy than following the harder rounding-up strategy. Moreover, they found that this was independent from the difficulty of problems, suggesting that the underlying cause of sequential difficulty effects can be the strategies elicited by problems.

Uittenhove and Lemaire suggested that strategy sequential difficulty effects (SSD effects) result from traces of a previous strategy execution lingering in working memory (WM). These traces interfere with the next strategy execution, which also requires WM resources. Since difficult strategies rely more on WM than easy strategies (Fürst & Hitch, 2000; Hitch, 1978; Imbo, Duverne, & Lemaire, 2007; Imbo & Vandierendonck, 2007; Logie, Gilhooly, & Wynn, 1994), Uittenhove and Lemaire (2012) assumed that less WM would be available following difficult strategies. Less functional WM capacities would slow down execution of the next strategy. In addition, traces from a previous difficult strategy in WM could interfere with information retrieval for the current strategy execution, by further demanding available WM capacities (see also Uittenhove, Poletti, Lemaire, & Dufau, in press).
In this study, we aimed at testing the link between WM and strategy sequential difficulty (SSD) effects. To do this, we tested whether SSD effects in arithmetic were correlated with individual differences in WM. If SSD effects indeed result from traces of a previous difficult strategy reducing functional WM capacities for the next strategy execution, we expect these effects to increase in individuals with fewer WM capacities. This can result from one or from several mechanisms. Individuals with low WM could be less able to keep multiple elements simultaneously active in WM, so that the presence of traces from a previous strategy takes up larger proportions of available capacities for the next strategy execution. Alternatively, or in addition, low-WM individuals could be less efficient in managing the content of their WM (e.g., suppression of no-longer relevant elements), so that traces of previous strategy execution in WM interfere for a longer duration with the next strategy execution.

In the present study, we also wanted to test whether giving participants more time between problems could reduce interference from execution of a previous difficult strategy with the current strategy execution. More time between problems could decrease lingering traces of the previous strategy in working memory through various mechanisms. For example, executive functions managing the content of WM would have more time to remove traces of no-longer relevant information (i.e., via a deletion inhibition mechanism; Hasher & Zacks, 1988). Alternatively, giving more time between problems could also lead to greater temporal decay of traces in WM (Barrouillet, De Paep, & Langerock, 2012). In either case, we should find that SSD effects can be remedied by giving participants more time between problems (see Uittenhove et al., in press for results showing that SSD effects only interacted with early stages of strategy execution, when less time had elapsed between problems).

We assessed individuals’ WM capacities with complex working-memory span tasks. These usually test how many items (words, digits, letters) a person can keep online for recall in a situation that requires resisting interference, shifting attention, and manipulating information. Note that this type of test involves not only information maintenance but also management of the content of WM. We assessed SSD effects in the same individuals by comparing solution latencies when participants were asked to provide estimates to two-digit addition problems (e.g., 32 + 68) by giving participants more time between problems (see Uittenhove et al., in press for results showing that SSD effects only interacted with early stages of strategy execution, when less time had elapsed between problems).

We assessed individuals’ WM capacities with complex working-memory span tasks. These usually test how many items (words, digits, letters) a person can keep online for recall in a situation that requires resisting interference, shifting attention, and manipulating information. Note that this type of test involves not only information maintenance but also management of the content of WM. We assessed SSD effects in the same individuals by comparing solution latencies when participants were asked to provide estimates to two-digit addition problems (e.g., 32 + 68) by giving participants more time between problems (see Uittenhove et al., in press for results showing that SSD effects only interacted with early stages of strategy execution, when less time had elapsed between problems).

2. Method

2.1. Participants

Sixty (15 men; mean age: 20 years; age range: 17–31 y.o.) undergraduates from Aix-Marseille Université (France) received course credit for their participation. The participants were unaware of the goal of the study. Two participants were excluded for extensive talking or coughing during the experiment, distorting the solution latencies.

2.2. Material

2.2.1. WM-tasks

WM-capacities were tested with three tasks: The operation span, the running span, and the reading span task. The operation span (Unsworth, Heitz, Schrock, & Engle, 2005) required participants to recall a series of 2 to 7 letters when these were alternated by arithmetic verification problems (e.g., "(4 + 2) − 1" followed by "A" and "true" or "false" followed by 1 to 6 similar problems and letters). The score for operation span was calculated as the total number of letters recalled correctly in all of the trials. The participants were told to focus on the arithmetic part and had to maintain above 80% accuracy in order for the results to be valid. The running span (Broadway & Engle, 2010) required participants to recall the last three to six letters of a series that contained either the same number or more letters (e.g., participants were asked to recall the last three letters of the following series: a g h t → a g h t). The score for running span was calculated as the total number of letters recalled correctly. The reading span (Daneman & Carpenter, 1980) required participants to recall the last words of a series of two to five sentences in the correct order while having to perform semantic judgment on the sentences (e.g., “Dans le lac nagent des saladiers” followed by “Correct?” or “Incorrect?” followed by 1 to 4 similar sentences). We used a French version of the task (Delaloye, Ludwig, Borella, Chicherio, & de Ribaujierre, 2007). The score for reading span was calculated as the total number of words recalled correctly in all of the trials.

2.2.2. Computational estimation task

We used the same stimuli as Uittenhove and Lemaire (2012). Two-digit addition problems (e.g., 32 + 68) were created. These problems were constructed so that a third of the problems were best estimated with the rounding-down (RD) strategy, with both operands including unit digits smaller than 5 (e.g., 43 + 64). Another third of addition problems were best estimated with the rounding-up (RU) strategy, with both operands including unit-digits larger than 5 (e.g., 47 + 68). The final third of problems were best solved with the mixed-rounding (MR) strategy with the unit digit of the first operand being smaller than 5 and the unit-digit of the second operand being larger than 5 (e.g., 43 + 69). Trials consisted of two problems each. Similar to Uittenhove and Lemaire (2012), experimental trials required execution of MR on the second problem. We thus presented participants with 32 trials in which MR on the second problem was preceded by RD (‘RD-MR’ trials) and 32 trials in which MR on the second problem was preceded by RU (‘RU-MR’ trials). However, to balance the design and neutralize the larger proportion of execution of MR, we added 16 ‘RD-RU’ and 16 ‘RU-RD’ filler trials. This ensured that each strategy was executed equally often. Only experimental trials were retained for data analysis.

Moreover, we controlled the sequence of strategy execution over longer series of items. We wanted to avoid sequential effects from previous strategy executions being different for the first strategy execution in one type of experimental trial than in the other type of experimental trial (e.g., execution of RD in ‘RD-MR’ trials being preceded by easier strategies than execution of RU in ‘RU-MR’ trials). We controlled this over three sequential trials (i.e., six problems) as follows (see Fig. 1): The first trial was always a filler trial (‘RD-RU’ or ‘RU-RD’), the second trial was one type of experimental trial (‘RD-MR’ and ‘RU-MR’), and the third trial was the other type of experimental trial (i.e., if ‘RD-MR’ was presented on the previous trial, ‘RU-MR’ was presented on the next trial). Each possible transition between these three trials was presented equally often. RD and RU strategy executions on the first problems in the experimental trials were thus each preceded by RD in 25% of cases, by RU in another 25% of cases, and in the remaining 50% by MR.
Moreover, following previous findings in arithmetic (see Campbell, 2005, for an overview), the following additional constraints were imposed on the selection of problems: (a) No operands included a repeated digit (e.g., 44) or a 0 or 5 digit (e.g., 20 or 25), (b) No problems included reversed operands (e.g., 43 + 82 and 82 + 43), (c) The first operand was larger than the second operand in half the problems, and smaller in the other problems, (d) No operands could be rounded to 0, 10, or 100, (e) The operands of the same problem could never be rounded to the same decade (e.g. 43 + 41), (f) The mean exact sums of problems were equal in all cells of the design, (g) The mean difference between exact sums and estimated sums of problems was equal in every cell of the design, (h) Half the problems involved a carry operation on decades (e.g., 64 + 73) in each cell of the design, (i) All transitions between problems involving carry operations and no-carry problems were presented equally often in all cells of the design, and (j) The estimated sums of successive problems were never the same (e.g., 43 + 72 followed by 32 + 83).

2.3. Procedure

All participants first completed the computational estimation task, and then the operation span, reading span, and running span tests.

2.3.1. Computational estimation

The stimuli were presented in a 72-point font on a 1280 × 800 computer screen. Participants were told that they were going to see addition problems to which they had to estimate the answer with one of three strategies. RD was explained as rounding both operands down to the smaller decades (e.g., 43 + 24 = 40 + 20 = 60); RU was described as rounding both operands up to the larger decades (e.g., 48 + 29 = 50 + 30 = 80); and MR was presented as rounding the first operand up to the larger decade and the second operand down to the smaller decade (e.g., 48 + 23 = 50 + 20 = 70). For each problem, the strategy to use was indicated by two arrows presented above the operands of the problem, with the direction of the arrows indicating how the operands had to be rounded (i.e., an arrow pointing down indicated that the operand had to be rounded down and an arrow pointing up indicated that the operand had to be rounded up). Each problem matched the indicated strategy. That is, the RD strategy was required on problems with unit digits smaller than 5; the RU strategy was to be executed on problems with unit digits larger than 5; and the MR strategy was imposed on problems including a first small unit-digit operand and a second large unit-digit operand. Participants were instructed to say the estimate of each problem out loud so as to control that they executed the required strategy. They saw two blocks of 48 trials (i.e., 96 problems per block and 192 problems total). One of the blocks consisted of trials with short RSI (300 ms) and the other block consisted of trials with long RSI (600 ms). In the block with short RSI, problems were separated by a 100-ms blank screen followed by a 100-ms fixation cross followed by another 100-ms blank screen. In the block with long RSI, problems were separated by a 200-ms blank screen followed by a 200-ms fixation cross followed by another 200-ms blank screen (see Fig. 2). The time until each response was measured by instructing participants to execute a concurrent key press when giving their verbal response to the problems.1 Errors were recorded by having the experimenter write down the answers of the participants so errors could later be identified. In the 41 participants included in the correlation study, we always presented the short RSI condition of the computational estimation task first, to avoid variations in SSD effects due to the order of presentation of RSI conditions. We tested 17 additional participants with the long RSI-block presented first to check whether there was an effect of order of presentation of RSI blocks on SSD effects.

3. Results

The first analysis was aimed at checking the relative difficulty of our strategies such that the rounding-down strategy yielded best and rounding-up worst performance, and whether this remained stable with RSI (relative strategy difficulty being a crucial factor to SSD effects). The second analysis aimed at testing SSD effects and the effects of short and long RSI. Before doing this analysis, we checked whether order of presentation of RSI interacted with SSD effects. These analyses revealed only an interaction between order of presentation of RSI and RSI on solution latencies, $F(1,56) = 22.8$, $MSe = 6,033,277$, $r^2 = 0.29$. Participants solved problems more slowly under short RSI (4860 ms) than under long RSI (4726 ms) conditions only when the short RSI was presented first, $F(1,56) = 52.9$, $MSe = 14,000,000$. There were no interactions between order and SSD effects, eliminating the need to include this variable in the analysis.

We also conducted exploratory analysis of the role of problem difficulty in SSD effects. Difficult problems required a carry operation on the tens (e.g., 80 + 30) whereas easy problems did not (e.g., 40 + 30). The transition between these problem types was controlled orthogonally to transition between strategies. However, analysis including problem difficulty is merely exploratory because of the low number of observations per cell (8).

The third and final analysis aimed at testing the correlation between participants’ SSD effects and their WM. Prior to analyses on solution latencies, values exceeding the mean + 2 × Standard Deviation (4%) and all trials containing an error (11.7%) were removed for each participant. All reported effects are significant with $p < .05$.

3.1. Relative strategy difficulty

We conducted a repeated measures ANOVA on participants’ mean solution times and percent errors on the first problem of each trial with strategy and RSI as within-participants variables (See Table 1). We found significant main effects of strategy and RSI, but no interaction effects between these two variables.

---

1 We used self-executed key press registering instead of voice-key registering because during calculation, participants had the tendency to verbalize or to make other unintentional sounds, setting off the voice key prematurely. We do not expect our measurement procedure to have induced systematic differences between conditions since it was applied in the same way on the same strategy in both conditions. Moreover, Lemaire and Lecacheur (2010) have compared voice-key data and experimenter-key press in the same type of experiment. They have found identical patterns of results with both measurement procedures.
Solution latencies varied with strategies, $F(1,157) = 104.8$, $MSe = 266,655$, $\eta^2_p = 0.65$. Participants were slower when executing the rounding-up strategy (4867 ms) than when executing the rounding-down strategy (4174 ms). Solution latencies also varied with RSI, $F(1,157) = 11.2$, $MSe = 495,587$, $\eta^2_p = 0.16$. Participants were slower when solving problems in the short RSI block (4675 ms) than in the long RSI block (4366 ms). The effect of strategy on solution latencies did not vary with RSI, $F < 1$.

Percent errors did not vary with strategies, $F < 1$, but varied with RSI, $F(1,157) = 14.7$, $MSe = 0.0034$, $\eta^2_p = 0.20$. Participants erred more when solving problems in the short RSI block (10.2%) than in the long RSI block (7.3%). The effect of strategy on percent errors did not vary with RSI, $F < 1.4$.

3.2. Strategy sequential difficulty effects

We conducted repeated-measures ANOVAs on solution times and percent errors on the second problem with the strategy on the first problem as a within-participants variable, in the short and long RSI blocks (see Table 2). Moreover, we conducted exploratory analyses on the role of problem difficulty in SSD effects (difficulty of first problem × difficulty of second problem × strategy used on first problem). We found main effects of the strategy used on the previous problem only in the short RSI condition. Regarding problem difficulty, we found main effects of the difficulty of the second problem as well as an interaction between difficulty of the first problem and SSD effects.

In the short RSI condition, participants were significantly slower after solving problems with the rounding-up strategy (4947 ms) than after the rounding-down strategy (4693 ms), $F(1,157) = 6.2$, $MSe = 1,872,162$. In the long RSI condition, the strategy used on the first problem had no effect on participants’ solution latencies on the second problem, $F < 1$. Analyses of mean percent errors revealed no effects ($F$-values < 2.1).

Including problem difficulty on the first and second problems of a trial, we found main effects of difficulty of the second problem, participants were faster when solving problems not involving carry (4204 ms vs 4818 ms), $F(1,140) = 51.1$, $MSe = 1,212,581$. We also found interaction effects, revealing that participants were faster following rounding down than when following rounding up with short RSI only when the first problem involved no carry, $F(1,140) = 3.8$, $MSe = 874,406$. When the first problem involved a carry, there was no difference after rounding down and rounding up, $F < 1$. Whereas solution latencies were equally fast after a difficult strategy on difficult problems than after a difficult strategy on easy problems (4755 ms vs 4679 ms), $F < 1$, solution latencies were shorter after a rounding-down strategy on easy problems than after a rounding-down strategy on difficult problems ($4470$ ms vs $4720$ ms), $F(1,40) = 5.2$, $MSe = 500,118$ (Fig. 3). In the long RSI condition, the effect of the strategy used on the first problem was not significant irrespective of the difficulty of the previous problem, $F_s < 1.3$.

3.3. WM capacities and SSD effects

We calculated SSD effects for each participant by taking the difference between the average solution latencies for the mixed-rounding strategy following rounding-up and following rounding-down divided by the mean solution latencies following rounding down. This yielded SSD effects as fractions of individuals’ mean solution latencies. For WM, for each individual, we calculated a $z$-score for each test. We performed a correlation test on participants’ SSD effects and the three measures of WM (See Table 3), leading to significant correlations.

SSD effects were negatively correlated with operation span ($r = −.29$, $p = .07$), reading span ($r = −.20$, $p = .21$), and running span ($r = −.22$, $p = .16$). The difference between these correlations was not significant, $p < .62$. The WM-capacities tests positively correlated amongst each other ($rs > .31$). Given that these three tests are assumed to measure the same underlying factor, we conducted a principal component analysis to determine the loadings of the three tests on a common component, captured by the first factor, that we assumed were WM capacities. The proportion of variance explained by the first factor was .58. We used these loadings to reconstitute a component score for each participant and found that it correlated significantly negatively with SSD effects, $r = −.31$, $p < .005$.

Finally, we performed an ANOVA with the strategy on the first problem as a within-participants variable and WM component scores (20 low and 21 high, with median WM score as the cut-off point) as a between-participants variable on second problem latencies. This test revealed that only individuals with low WM had significant SSD effects ($415 ms, F = 13.4 vs 29 ms, F < 1$ in high-WM individuals), $F(1,39) = 5.9$, $MSe = 128,550$, $\eta^2_p = 0.13$. Consequently, only low-WM individuals showed significant SSD effect decreases with long RSI (654 vs. 176 ms), $F = 5.6$, whereas high-WM individuals did not (27 ms vs 32 ms), $F < 1$.

4. Discussion

Schmeichel (2007) found that efforts at executive control temporarily undermined subsequent efforts at executive control. For example,
Another interesting finding from this study was that SSD effects disappeared with increased RSI. This suggests that giving participants sufficient time between problems reduces traces of previous strategy executions in WM interfering with next strategy executions. This latter finding suggests that our effects differ in important aspects from the effects reported by Schmeichel (2007). Indeed, contrary to Schmeichel, our effects were present for a brief time only. This suggests that different mechanisms underlie both effects. Whereas Schmeichel's effects could be caused by long-lasting working-memory fatigue, our effects could be due to residual occupation of working memory by a previous difficult strategy, with a much more transient time course. We see three major ways in which traces in WM could degrade with increasing RSI. The first way is consistent with our suggestion that WM managing components are involved in SSD effects. With more time, participants would more efficiently inhibit no-longer relevant traces in WM (e.g., deletion inhibition, Hasher & Zacks, 1988). Second, traces could undergo temporal decay (without interference) in WM with increasing RSI (Barrouillet et al., 2012). Third, increasing RSI could allow interference from unrelated information (e.g., participants thoughts), so that traces in WM get overwritten similarly after execution of easy and difficult strategies.

A final question concerns the role of problem difficulty in the sequential difficulty effects found here. Uittenhove and Lemaire (2012) previously investigated this issue and found strategy sequential difficulty effects independently of problem difficulty. However, they manipulated problem difficulty in blocks (i.e., they gave participants a block of problems involving carry operations and a block of problems involving no carry operations). Presenting carry and no carry operations in blocks could have led to better preparation of carry operations than in the current study, neutralizing sequential difficulty effects related to carry operations. In the present study, carry and no-carry problems were mixed. Moreover, the transition between carry and no-carry problems was controlled. The permitted analyses of the role of problem difficulty, and this revealed that SSD effects interacted with problem difficulty. Whenever the previous problem is difficult, or the previous strategy is difficult, we are slower with the next strategy execution than when following an easy problem on which we used an easy strategy. This suggests that sequential difficulty effects can be carried over from problems to strategies and vice versa. Note however that although our design permitted exploratory testing of these interactions, we did not have sufficient trials in each cell of the design to draw strong conclusions. Future studies should be designed so as to measure interaction between SSD effects and problem difficulty with more observations per cell for more powerful tests.

Our results have implications for inter-individual differences in strategy execution: Individuals or populations suffering from declines in WM (e.g., older adults and AD patients) can be expected to suffer more from SSD effects (see for example Uittenhove & Lemaire, in press), which may hamper their problem-solving performance. These populations could be expected to need more time between problems to ensure optimal performance.

Our findings also have implications for how strategies are executed. Models of strategies (Strategy Choice And Discovery Simulation, Siegler & Arraya, 2005; Represent Construct Choose Learn, Lovett & Schunn, 1999 and Strategy Selection Learning; Rieskamp & Otto, 2006) explain strategy performance as a result of the number and type of procedures included in each strategy (i.e., more procedures or harder procedures in one strategy result in longer latencies). This is because execution of these procedures relies on limited WM resources (DeStefano & LeFevre, 2004). The present data suggest that WM resources also dynamically vary as a function of the difficulty of the strategy executed just before. Including a parameter for these resources within currently available computational models would enable these models to account for and to simulate strategy sequential difficulty effects and their link to WM.

Table 3

<table>
<thead>
<tr>
<th></th>
<th>SSDE</th>
<th>Operation span</th>
<th>Reading span</th>
<th>Running span</th>
<th>WM-component</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSDE</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Operation span</td>
<td>–0.29</td>
<td>–</td>
<td>–0.20</td>
<td>0.32*</td>
<td>–</td>
</tr>
<tr>
<td>Reading span</td>
<td>–0.20</td>
<td>0.32*</td>
<td>0.48***</td>
<td>0.32*</td>
<td>–</td>
</tr>
<tr>
<td>Running span</td>
<td>–0.31*</td>
<td>0.80**</td>
<td>0.68**</td>
<td>0.80**</td>
<td>–</td>
</tr>
</tbody>
</table>

Note: SSD = [RT mixed rounding following rounding up] – [RT mixed rounding following rounding down] / [RT mixed rounding following rounding down].

* p < .05.
** p < .01.

![Fig. 3. Effect of strategy and difficulty on the previous problem on solution latencies with mixed rounding on the next problem (solution latencies in ms and error bars).](image-url)
References