Children’s strategies in computational estimation

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Abstract

We investigated strategies used to estimate answers to addition problems. Two hundred and sixteen participants (72 adults, 72 sixth graders, and 72 fourth graders) had to provide estimates of three-by-three digit addition problems (e.g., 249 + 743). The choice/no-choice method was used to obtain unbiased estimates of the performance characteristics of strategies. Results showed that (a) at all ages, the most common strategy was to round both operands down to the closest smaller decades, (b) strategy use and execution were influenced by participants’ age, problem features, and relative strategy performance, and (c) age-related changes in computational estimation include changes in relative strategy use and execution, as well as in the relative influences of problem and strategy characteristics on strategy choices. Implications of these findings for understanding age-related differences in strategic aspects of computational estimation performance are discussed. © 2002 Elsevier Science (USA). All rights reserved.

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The goal of the present work was to understand age-related differences in strategic aspects of children's computational estimation skills. Computational estimation is defined as finding an approximate answer to arithmetic problems without actually (or before) computing the exact answer (e.g., \(146 + 69 + 48 = 260; 47 \times 53 = 2500\)). Investigating cognitive processes involved in computational estimation skill provides information about children's general understanding of mathematical concepts, relationships, and strategies (e.g., Bestgen, Reys, Rybolt, & Wyatt, 1980; Carpenter, Coburn, Reys, & Wislon, 1976; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Lemaire, Lecacheur, & Farioli, 2000; Sowder, 1988, 1992; Sowder & Wheeler, 1989). Not only is computational estimation an important component of mathematical cognition, but it is also often used in everyday situations in which a rough answer provides a contextually appropriate degree of precision. Examples of such situations involve (a) determining how much a dinner that cost 150 French Francs (or 22.5 Euros) would cost in US dollars, (b) converting the height of a 3000-m mountain to feet, or (c) converting a temperature presented in degrees Fahrenheit to the corresponding temperature in degrees centigrade (e.g., Carpenter et al., 1976; Rubenstein, 1985; Trafton, 1978).

To understand age-related differences in children's computational skills, we investigated strategic aspects of their performance. More specifically, the data document age-related changes in strategy use (Which of the available strategies children prefer? Do these preferences change with age and other factors? What are the determinants of strategy choices?) and strategy execution. (How fast and good are estimates with each available strategy? How do speed and precision of estimates change with age?) We use the choice/no-choice method devised by Siegler and Lemaire (1997) to understand these age-related differences while controlling for potential artifacts. Implications of the data are important for understanding both general aspects of children age-related differences in strategic behaviors and specific aspects of computational estimation. Before outlining the logic of the present work, we briefly review previous findings on children’s estimation computational skills.

Computational estimation has been studied in adults of varying computational proficiencies (Dowker, 1997; Dowker, Flood, Griffiths, Harriss, & Hook, 1996; Levine, 1982; Pelham, Sumarta, & Myaskovsky, 1994), as well as in children and adolescents (Baroody, 1989; Case & Sowder, 1990; Dowker, 1997; LeFevre, Greenham, & Waheed, 1993; Lemaire et al., 2000; Newman & Berger, 1984; Reys, Rybolt, Bestgen, & Wyatt, 1982; Sowder & Markovits, 1990). In most of these studies, computational estimation is investigated by asking participants to provide estimates (or approximate solutions) to arithmetic problems (e.g., \(457 + 349 = 800\)). All previous studies collected precision of estimates (as measured by the absolute or relative difference between estimates and correct answers) and verbal protocols
(i.e., people are asked to say how they found the solution), except in the Lemaire et al. study in which solution times were also collected.

In all the previously cited researches, participants used several computational estimation strategies. A strategy can be defined as “a procedure or a set of procedures to achieve a higher-level goal or task” (Lemaire & Reder, 1999, p. 365). In computational estimation, Levine (1982) found that adults rounded both operands and calculated $820 \times 30$ to estimate $825 \times 26$ or rounded up only one operand (such as when doing $93 \times 20$ to estimate $93 \times 18$). Children also use several strategies. For example, LeFevre et al. (1993) found that children as young as 9 years old estimated the $78 \times 189$ product by, among others, rounding both numbers ($80 \times 200$) or rounding only one number ($100 \times 189$).

Computational estimation strategies have been found to be used with different proportions in different age groups. For example, LeFevre et al. reported that children used so-called prior-compensation strategies (e.g., they would do $75 \times 200$ to estimate $78 \times 189$) more frequently than post-compensation strategies (e.g., they would do $80 \times 200 - (2 \times 200)$ to estimate $78 \times 189$) whereas adults did the reverse. In children, prior-compensation strategies were used more often by sixth graders than by eighth graders, while post-compensation strategies were used equally often in both age groups (see Dowker, 1997, for similar age-related differences in computational estimation task with addition problems). Finally, strategy use was found to differ with problem characteristics. Again in LeFevre et al.’s study, rounding both numbers occurred more frequently on larger problems (e.g., $36 \times 146$) than on smaller problems (e.g., $8 \times 112$).

Computational estimation strategies have been found to vary in efficacy and this strategy efficacy to improve with age. Some strategies yield greater speed and better estimates than others. For example, when they asked 10-year-old children to provide estimates of three-by-three digit addition problems, Lemaire et al. (2000) found that rounding with decomposition (e.g., doing $400 + 300 + 60 + 60 = 820$ to estimate $459 + 356$) yielded better estimates than truncation (e.g., doing $450 + 350 = 800$) and that children were quicker when using truncation than when using rounding with decomposition. All investigators comparing children of different ages have found increased precision of estimates with age. Although it is known that children retrieve correct answers to arithmetic problems more rapidly with age (e.g., Ashcraft & Koshmider, 1991; Geary, 1996; Lemaire, Barrett, Fayol, & Abdi, 1994; Lemaire & Siegler, 1995), the speed with which children estimate answers has not been assessed previously. Thus, the nature of age-related change in speed of estimation is unknown.

In sum, previous works on children’s computational estimation skills have found that, like in most cognitive domains (a) children use several strategies to find estimates of arithmetic problems, (b) these strategies vary
in frequency and efficacy, and (c) computational estimation strategy use and execution change with age.

Previous works on strategic aspects of children’s computational estimation were limited in several respects. First, as strategies are used unequally often by participants of different groups and on different types of problems, strategy use and execution were confounded. This makes it impossible to compare strategy use and execution across different groups of children and to know whether relative strategy performance stems from true differences in strategy speed and accuracy or from strategy selection (over items or participants) artifacts. Second, even though LeFevre et al. (1993) and Lemaire et al. (2000) have observed that computational estimation strategy choices were influenced by problem characteristics (such as the size of unit digits), we do not know whether children are adaptive in their strategy choices. Adaptive strategy choices are seen, among others, in strategy use correlating significantly with problem characteristics or with relative strategy performance. Increased correlation between strategy use and problem or strategy characteristics is the signature of age-related improvements in strategy selection mechanisms. Because of strategy use being confounded with strategy execution in previous research on children’s computational estimation, we do not know whether adaptiveness of computational estimation strategy choices improves with age.

One of the goals of the present study was to independently assess age-related changes in computational estimation strategy use and execution. To achieve this end, we used the choice/no-choice method devised by Siegler and Lemaire (1997) (see also Geary, Hamson, & Hoard, 2000; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Montani, 1997). The choice/no-choice method requires collecting performance under two types of conditions: (a) a choice condition in which participants are free to choose whichever strategies are available on each trial and (b) no-choice conditions in which participants must use a given strategy on all items. If there are two available strategies, there will be two subconditions in the no-choice conditions—one for each strategy. Unbiased estimates of strategy characteristics (for the sample that generated the performance) can be obtained when a given strategy is required for all participants on all trials. This makes it possible to objectively assess relative strategy speed and accuracy, compare strategy speed and accuracy across different age groups, and examine whether strategy preferences are associated with relative strategy benefits. That is, children may prefer one strategy over the other because it is easier to execute, which would result in greater speed or accuracy when children use that strategy. Comparison of strategy use and execution in participants of different ages can be made, without running the risk of confounding strategy use and strategy execution, and the interactions of these two aspects with participants’ age.

Two groups of children, one each from fourth and sixth grades, and a group of young adults had to provide estimates of 72 three-by-three digit
addition problems (e.g., 641 + 237). Fourth- and fifth-grade children have been tested, as previous studies suggest that the most important changes in strategic aspects of children’s computational estimation performance may occur between fourth and sixth grades (Baroody, 1989; Dowker, 1997; LeFevre et al., 1993; Lemaire et al., 2000). Participants were tested either in choice or in one of the no-choice conditions. In choice conditions, participants were allowed to choose between round-down strategy (i.e., rounding both operands to the closest smaller decade) and round-up strategy (i.e., rounding both operands to the closest larger decade) on each of the 72 experimental problems. (Choice/no-choice was a between-subject factor so that prior participation in the no-choice condition could not influence performance in the choice condition.) Participants in the no-choice/round-down strategy condition were required to use the round-down strategy on all 72 experimental problems, whereas participants in the no-choice/round-up strategy condition were required to use the round-up strategy on the 72 problems.

We did not test the so-called mixed strategy (i.e., round one operand down and the other up) and focused on round-down and -up strategies for the following reasons: (a) round-up and -down strategies are known to be used by children as young as 9 years old; (b) round-up and -down strategies involve common mechanisms (e.g., encoding operands, rounding both operands, finding an approximate sum and providing the answer aloud) enabling us to collect data on comparable strategies; they also involve different mechanisms (e.g., round-off involves incrementing decade digits), leading to easily understandable sources of differences in strategy use and execution. Finally, we wished to avoid the possibility of (c) an experimental situation leading to too strong strategy-problem feature associations. Such a situation in which children would have systematically chosen mixed strategy on mixed problems, round-down on small-unit problems, and round-up on large-unit problems, may have hidden potential age-related differences in the determinants of strategy choices other than problem features, one of the main interests of this study.

The data collected in this study enabled us to address the following issues regarding age-related changes in strategic dimensions of computational estimation: Do participants have preferences among computational round-down and -up strategies? If yes, are these preferences justified by strategy execution (i.e., speed, precision) characteristics? Are strategy use influenced by participants’ age and problem characteristics (i.e., distance between operands and closest decades)? Participants’ strategy preferences should be reflected in higher percentages of use of the most preferred strategy in choice conditions. This may be accompanied by greater strategy performance in no-choice conditions for the favorite strategy. Effects of problem features should be reflected in correlations between these features and percent use of strategies. Finally, age-related differences in strategy use and execution predict different strategy preferences in each age group,
increased strategy performance in older participants, and age-related differential influences of problem features and relative strategy performance on strategy use and execution.

Method

Participants

Two hundred sixteen individuals, 72 adults (27 males, 45 females), 72 fourth-grade children (38 boys and 34 girls), and 72 sixth-grade children (42 boys and 30 girls), participated in the study. The adults were undergraduate college students who received credit for participating in the experiment. Their mean age was 21.5 years.months (range = 18.3–22.5). The fourth-grade children had a mean age of 9.7 years (range = 8.9–10.0) and the sixth-grade children were 11.9 years of age on average (range = 11.3–12.3). Children were from a French upper-class urban public school.

Stimuli

The stimuli were 72 addition problems presented in a standard form (i.e., \(a + b\)), in which \(a\) and \(b\) were three-digit numbers with a mean correct sum of 865 ranging from 716 to 986. These 72 problems were selected so that across all problems mean estimates provided by round-down were not statistically different from those provided by round-up (mean difference between percent deviations between estimates yielded by each strategy was 0), so as not to trigger strategy preferences on the basis of overall differences in strategy precision.

Based on previous findings from Lemaire et al. (2000) and from LeFevre et al. (1993) regarding the effects of unit size on computational estimation performance, half of the problems in the present experiment were small-unit problems (i.e., problems made of operands with unit digits that were small) and half were large-unit problems (i.e., problems made of operands with unit digits of large magnitude). The correct sums of unit digits of both operands were 6.2 (SD = 2.00; range = 3–9) on average for small-unit problems and 13.8 (SD = 1.97, range = 11–17) for large-unit problems. For example, problems like 263+471 and 429+287 were small- and large-unit problems, respectively, because the sum of unit digits was 3+1 = 4 in the former and 9+7 = 16 in the latter.

Small- and large-unit problems were matched on the side of the larger operand, on the side of the operand with the smallest unit digit, and on the correct sum of both operands. The larger of the two operands was on the left position (e.g., 514 + 353) in half the problems and on the right position (e.g., 263 + 471) in the other problems; and the operand with the smallest
unit digits was on the left position (e.g., 527 + 232) in half the problems and on the right position (e.g., 582 + 174) in the other problems. Small-unit problems had a mean sum of 858 (SD = 62.2) ranging from 734 to 976 and large-unit problems had a mean sum of 871 (SD = 70.2) ranging from 716 to 986. Moreover, given certain experimental effects that are known in the domain of mental arithmetic (see Dehaene, 1997; Geary, 1996, for reviews), we controlled the following factors: (a) no operand had 0 or 5 as unit digits; (b) digits were not repeated in the same unit, decade, or hundred positions across operands (like in 346 + 149 or 346 + 376); (c) no digits were repeated within operands (i.e., like in 344 + 659); (d) no reverse orders of operands were used (i.e., if 383 + 458 was used, 458 + 383 was not used); and (c) carry-over operations involved with one strategy were also involved when using the other strategy (e.g., 473 + 364 involved carry-over operations with both round-down and -up strategies).

Procedure

Before encountering the experimental problems, participants were told that they would be asked to give an approximate answer to an arithmetic problem that is as close as possible to the correct answer, without actually calculating the correct answer. An example was worked out with participants who were told “for example, if I have to estimate 138 + 456, I can do 140 + 460, and give 600 as an approximate solution to the problem. I can also do 130 + 450 and give 580 as an answer, or do anything else that yields an approximate answer.” Then, all participants were told “you are going to see 72 problems such as 138 + 456, and your task is to tell me an approximate sum.” Participants in the choice condition were also said “To estimate these 72 sums, you can use either round-up or round-down strategies, and no other strategies. Round-down strategy means that you round each operand to the closest smaller decade, like when you do 130 + 450 to estimate 138 + 456. Round-up strategy means that you round each operand up to the closest larger decade, like when you do 140 + 460 to estimate 138 + 456.” Participants in the no-choice/round-up strategy condition were required to use the round-up strategy on all problems and participants in the no-choice/round-down strategy condition were required to use the round-down strategy on all problems. After an initial practice period, all individuals had no difficulties with either round-down or round-up strategies (in the no-choice conditions) or both (in the choice condition). In the beginning of practice, most of the participants wanted to use both rounding strategies on the same problems (i.e., rounding down the first operand and rounding up the second operand on problems like 423 + 356). However, after a while, all participants understood that this strategy was not allowed.

The experimental problems were presented in 60-point bold Palatino font in the center of a 14-in. computer screen controlled by a Power Macintosh
PC 1400. Each trial began when a 1000-ms ready signal (the word “prêt?” for “ready” in French) appeared in the center of the screen. Then, the three-by-three digit addition problems were displayed horizontally in the center of the screen. The symbol and numbers were separated by spaces equal to the width of one character. Timing of each trial began when the problem appeared on the screen and ended when the experimenter pressed a button on a button box, the latter event occurring as soon as possible after the participant’s response. The software [PsyScope (Cohen, MacWhinney, Flatt, & Provost, 1995)] collected data with 1-ms accuracy. To stress the estimation nature of the expected response, we asked participants to be as fast and as close to correct sums as possible. After each response in the choice condition, participants were asked “Which of the two strategies did you use, round-down or round-up strategy?” On each trial, the experimenter recorded participants’ responses (in both choice and no-choice conditions) and verbal protocols (in the choice condition). In the choice condition, problems remained on the screen during verbal protocols as pilot testing revealed that this made it easier for participants to describe their strategy.

The order of presentation of problems was randomized for each participant. Each participant was permitted a 5–10 min rest between blocks of 36 problems each. Before the experimental trials, participants were given 10 practice problems to familiarize themselves with the apparatus, procedure, and task.

Experimental conditions

Participants solved estimation problems under one of the three conditions, choice, no-choice/round-down, and no-choice/round-up conditions. In the choice condition, participants could round both operands up or off to the closest decades to find an estimate on each problem. In the no-choice/round-down, participants were required to round both operands down to the closest smaller decades to generate an answer to all problems (e.g., 263 + 471 = 260 + 470 = 730). In the no-choice/round-up condition, participants were required to round both operands up to the closest larger decades to find an estimate (e.g., 429 + 287 = 430 + 290 = 720) to all problems.

Results and discussion

Results are reported in two main parts. The first analyzes age-related differences in strategy use and the second examines age-related differences in strategy execution. In all results, unless otherwise noted, differences are significant to at least \( p < .05 \). ANOVAs were run with participants (\( F_1 \)) and with problems (\( F_2 \)) as random factors to control for potential item-specificity effects.
**Age-related differences in strategy use**

This section analyzes strategy use in the choice condition. First, we analyze strategy variability. More precisely, we ask whether participants used both strategies (or only one), whether both strategies were used equally often, and whether strategy use was influenced by the size of unit digits in operands. Second, we analyze adaptivity of strategy use. More precisely, we discuss correlations between strategy use, on the one hand, and problem and strategy characteristics, on the other hand. We also analyze the predictors of strategy use in each age group so as to document age-related changes in the determiners of strategy choices.

*Variability in strategy use*

The first question concerning strategy use was whether participants accomplished the computational estimation task with only one strategy or with both strategies. In other words, were there any within-participants variability in strategy use? One way to answer this question is to tally the number of participants using round-down on each of the three different ranges of problems so as to know how many participants used only one strategy and how many participants used round-down (or round-up) most often. None of the participants used round-down on less than 34% of problems; 18 adults, 19 sixth graders, and 16 fourth-grade children used it on between 34% and 66% of problems; and 6 adults, 5 sixth graders, and 8 fourth graders used it on more than 66% of problems. Similarly, analysis of variability in strategy use across items revealed that no problems were estimated with round-down by less than 34% of participants; 41, 50, and 43 problems were solved with round-down between 34% and 66% of participants in adults, sixth, and fourth graders, respectively; and that 31, 22, and 29 problems were solved with round-down by more than 66% of adults, sixth, and fourth graders, respectively.

After verifying that strategy variability was not the result of averaging effects, we analyzed mean percentages of use of round-down strategy with analyses of variance (ANOVAs) involving a 3 (group: adults, sixth, and fourth graders) × 2 (unit size: small, large) design, with age as the only between-subject factor.

As can be seen from Fig. 1, participants of all age groups used round-down most often (61%). The main effect of age was marginally significant only when problems were entered as a random factor, $F(1, 69) = 1.15$, MSE = 162.48; $F(1, 140) = 2.73$, $p = .06$, MSE = 102.42, as fourth graders (59%) tended to use round-down less often than sixth graders (62%) or adults (63%). Round-down was used overall more often with small-unit problems (68%) than with large-unit problems (54%), $F(1, 69) = 25.37$, MSE = 547.35; $F(1, 68) = 81.43$, MSE = 127.93. More interestingly, the
age \times \text{unit size interaction}, \ F(1, 23) = 13.33, \ MSe = 356.84; \ F(1, 70) = 62.92, \ MSe = 113.37 \) and in sixth graders (70\% vs. 54\%; \ F(1, 23) = 14.37, \ MSe = 219.29; \ F(1, 70) = 37.53, \ MSe = 125.92), but not in fourth-grade children (61\% vs. 56\%; \ F(1, 23) = 1.23, \ ns, \ MSe = 244.90; \ F(1, 70) < 1).

In sum, analyses of strategy variability revealed that participants of all age groups used both round-down and -up strategies to accomplish the computational estimation task and favored the round-down strategy. These analyses also revealed that the round-down strategy was used somewhat more often by sixth-grade children and adults, and more so when problems involved operands with units smaller than 5.

Adaptivity in strategy use

Previous works on strategy use showed that adults and children adapt the strategies they use to accomplish a cognitive task to two types of parameters, namely problem and strategy characteristics (see Siegler, 1996, for a review). Problem characteristics refer to a variety of features, such as problem size, whether operands include specific numbers like 0, 1, 5, or 10, or side of the larger operand. Strategy characteristics refer to relative strategy performance with a given strategy. For example, Siegler and Lemaire (1997) have shown that strategy choices to solve complex multiplication problems were predicted by both relative strategy speed and whether one of the multiplicands was 10. The goals of these analyses were to determine (a) whether computational estimation strategy use is influenced by
both problem and strategy characteristics and (b) which of the problem and strategy characteristics is the best predictor of computational estimation strategy use. We also ask whether, once one of these characteristics is taken into account, the other feature exerts an independent effect on percent use of round-down.

First, to know whether participants adjusted their strategy use to problem and/or strategy characteristics, we ran a series of correlation analyses. Problem-based correlations in each age group were calculated between mean percent use of round-down on each problem and two types of variables. One set of variables characterized problem features and the other relative strategy performance. Problem features included (a) correct sums (e.g., 716 for 429 + 297), (b) sum of hundred and decade digits (e.g., 17 for 429 + 297), and (c) sum of unit digits (e.g., 16 for 429 + 297). Relative strategy performance included (d) relative strategy precision (i.e., how good are estimates yielded by each strategy), and (e) relative strategy speed (i.e., how fast participants are with strategies). Relative strategy precision was calculated for each problem in two steps. First, mean percent deviations between estimates and correct sums were calculated for each strategy and each problem. Second, differences between these mean percent deviations were calculated. Thus, the formula for relative strategy precision was \( \frac{\text{round-up/no-choice mean percentage deviations between estimates and correct sums} - \text{round-down/no-choice mean percentage deviations between estimates and correct sums}}{\text{round-up/no-choice mean percentage deviations between estimates and correct sums}} \). Similarly, relative strategy speed was calculated with the following formula: \( \frac{\text{RTs for round-up/no-choice} - \text{RTs for round-down/no-choice}}{\text{RTs for round-up/no-choice}} \). Significant correlations between mean percent use of round-down and problem or strategy characteristics were expected as the signatures of participants’ strategy use being influenced by these two types of variables. The correlation matrix is presented in Table 1.

Overall, participants’ strategy use was influenced by problem characteristics and relative strategy performance. However, the influence of these variables was not the same in each age group. Correlations between mean percent use of round-down and sum of unit digits were equally high in adults and sixth-grade children, \( t(70) = 1.54 \text{ ns} \), such that both adults and sixth graders tended to use round-down less often as the size of units increased. These correlations between mean percent use of round-down and sum of unit digits were smaller in fourth-grade children than in adults, \( t(70) = 4.49 \), or in sixth-grade children, \( t(70) = 2.95 \). Fourth-grade children were more heavily influenced by sum of hundred and decade digits \( r = .36 \), using round-down more often as this sum increased.

The second important set of findings to note in these correlations concerns people’s calibrations of strategy use to relative strategy performance, as measured by correlations between mean percent use of round-down and relative strategy precision. Here again, correlations were not the same in
sixth-grade and adult participants, on the one hand, and in fourth graders, on the other hand. Correlations between mean percent use of round-down and relative strategy precision were larger in adults than in sixth graders, $t(70) = 1.94$, and than in fourth graders, $t(70) = 3.22$; it was marginally larger in sixth- than in fourth-grade children, $t(70) = 1.38$, $p < .10$. Adults and sixth graders used round-down more often when it yielded estimates that were closer to correct answers. Moreover, mean percent use of round-down tended to slightly correlate with relative strategy speed only in adults ($r = .23$), as if they tended to use round-down when it yielded faster solution times. Note that, contrary to adults and sixth graders, fourth graders’ use of round-down did not correlate with any relative strategy performance measure.

In sum, these correlation analyses suggest that adults and sixth graders chose strategies on individual problems based on distances between unit digits and their closest decades, as well as based on relative strategy precision. By contrast, fourth graders made strategy choices mostly on the basis of the size of hundred and decade digits and to a much lower extent on the size of unit digits. Size of hundred and decade digits captures calculation

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<th>Variable</th>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tr>
<td>1. Percent use of round-down</td>
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<td>-.13</td>
<td>-.78**</td>
<td>.55**</td>
<td>.23*</td>
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<td>.06</td>
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<td>3. Sum of hundred and decade digits</td>
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<td>-.05</td>
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<td>4. Sum of unit digits</td>
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<td>-.30</td>
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<td>5. Relative strategy precision</td>
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<td>6. Relative strategy speed</td>
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<td><strong>Sixth-grade children</strong></td>
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<td>.00</td>
<td>-.64**</td>
<td>.28*</td>
<td>-.03</td>
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<td>2. Correct sum</td>
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<td>-.03</td>
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<td>5. Relative strategy precision</td>
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<td>6. Relative strategy speed</td>
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<td><strong>Fourth-grade children</strong></td>
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<tr>
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<td>2. Correct sum</td>
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</tr>
<tr>
<td>3. Sum of hundred and decade digits</td>
<td>-.09</td>
<td>.08</td>
<td>.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Sum of unit digits</td>
<td>.12</td>
<td>-.12</td>
<td></td>
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<td></td>
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<tr>
<td>5. Relative strategy precision</td>
<td></td>
<td></td>
<td></td>
<td>-1.0</td>
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<tr>
<td>6. Relative strategy speed</td>
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*p < .05.

**p < .01.
difficulty, suggesting that fourth graders used round-down more often on problems involving large numbers to decrease calculation difficulty. Previous research in arithmetic showed that younger children have more difficulties with multi-digit additions, and in particular, in processing hundred and decade values of numbers in multi-digit operands (e.g., Fuson, 1986, 1990; Fuson & Briars, 1990).

These results were confirmed in stepwise regression analyses conducted separately in each age group to determine which of the two types of—problem or strategy characteristics—variables best predicts the mean percent use of round-down on each problem. Results showed a single significant predictor in all age groups. This best predictor of strategy use was the sum of unit digits in adults \( r^2 = .60 \) and in sixth graders \( r^2 = .41 \). Fourth graders’ strategy use was best predicted by sum of hundred and decade digits \( r^2 = .13 \). Note that, in addition to age differences in the type of best predictors, differences in \( r^2 \) magnitudes suggest that younger participants were much less systematic in their strategy use. Correlation between fourth graders’ use of round-down and sum of hundred and decade digits was significantly lower than those correlations in adults, \( t(70) = 3.77 \), or sixth graders, \( t(70) = 2.24 \).

Although differences between mean percent deviations for each strategy significantly correlated with mean percent use of round-down in adults and sixth graders, it did not do so in the regression analyses. The reason was that all of the variance captured by this variable was also captured by sum of unit digits and the latter variable was considerably more highly correlated with the percentage of use of round-down on each problem. Considered by itself, difference in mean percent deviations accounted for 30% and 8% of the variances in adults’ and sixth graders’ percentages of choices of round-down on each problem. Although these were significant, sum of unit digits accounted for an additional 30% and 33% of variance in adults and sixth graders, respectively.

**Age-related differences in strategy execution**

This section analyzes strategy execution in both the choice and no-choice conditions. Based on previous works on computational estimation, we first tested effects of age and unit size on strategy execution. Then, we analyzed one of the most robust effects in the arithmetic literature (see Ashcraft, 1995 for a review), that is, problem-size effects. Finally, we examined whether having the choice facilitates participants’ performance. Strategy execution was analyzed in terms of speed and accuracy of participants’ estimates. Following previous works on computational estimation, accuracy of participants’ estimates was assessed by calculating percent deviations between estimates and correct sums for each problem and each participant (e.g., LeFevre et al., 1993; Lemaire et al., 2000; Levine, 1982). To illustrate,
suppose a participant responding 570 for 132 + 453. That participant would be 2.56% (([570 − 585]/585) × 100) away from the correct sum.¹

Effects of age and unit size on strategy execution

Mean solution latencies² and percent deviations in each choice and no-choice conditions were analyzed with ANOVAs. Choice mean solution times and percent deviations were collapsed over problem types, as there were too many missing cells resulting from strategy choices. Therefore, ANOVAs in choice performance involved a 3 (group: adults, sixth, and fourth graders) × 2 (strategy: round-down, round-up) design, with repeated measures on strategy. ANOVAs of no-choice mean latencies and percent deviations were run with a 3 (group: adults, sixth, and fourth graders) × 2 (strategy: round-down, round-up) × 2 (unit size: small, large), with repeated measures on the last factor. Mean solution times and percent deviations for each estimation strategy in each age group are presented in Figs. 2 and 3, separately in the choice and no-choice conditions.

Analysis of choice-condition speed showed significant main effects of age, strategy, and interaction effect of age × strategy. As can be seen in Fig. 2, adults (6.6 s) were significantly faster than sixth graders (12.8 s) who were significantly faster than fourth graders (17.5 s), $F(1, 69) = 48.49$, $MSe = 29.98$; $F(2, 142) = 961.11$, $MSe = 4.59$; and round-down was 2.6 s faster than round-up, $F(1, 69) = 52.99$, $MSe = 2.67$; $F(1, 71) = 121.69$, $MSe = 5.39$. The age × strategy interaction was significant only in problem-based analyses, $F(1, 69) = 1.61$, $p = .21$, $MSe = 2.67$; $F(2, 142) = 5.25$, $MSe = 4.08$. These latency results were not compromised by speed–accuracy trade-offs, as corresponding analyses on mean percent deviations showed significant effects of age only, with adults (1.52%) and sixth-grade children (1.54%) providing estimates that were closer to correct sums than fourth graders (2.06%); $F(1, 69) = 8.98$, $MSe = 0.44$; $F(2, 142) = 12.03$, $MSe = 0.78$. However, these effects in the choice conditions should be interpreted with caution because they result from both strategy choices and strategy execution characteristics. The no-choice condition provides unbiased measures of strategy performance and allows analyses of the role of problem characteristics on strategy execution.

In the no-choice condition, each strategy was required on all problems. Performance in this condition yielded estimates of strategy performance that

¹ We also calculated deviations between estimates and correct sums given the strategy that was used. For example, the participant responding 570 for 132 + 453 after using the round-down strategy would be 1.72% away from the correct sum yielded by the round-down strategy. Both types of percent deviations yielded exactly the same effects in analyses of strategy execution.

² All the reported analyses were also run with medians and yielded the same outcomes.
Fig. 2. Mean solution times (in s) for round-down (RD) and round-up (RU) strategies in each group of participants under choice and no-choice conditions.
Fig. 3. Mean percent deviations for round-down (RD) and round-up (RU) strategies in each group of participants under choice and no-choice condition.
were not biased by selective use of the strategies on different problems and by different individuals. As can be seen from Fig. 3, adults (5.6 s) were faster than sixth graders (10.1 s) who were faster than fourth graders (16.4 s), \( F_1(2, 138) = 88.81, MSe = 63.47; F_2(2, 136) = 1490.50, MSe = 2.89 \). Round-down was 4.9 s faster than round up, \( F_1(1, 138) = 53.56, MSe = 63.47; F_2(1, 68) = 2200.15, MSe = 1.14 \). This difference in strategy speed was larger in fourth graders (means were 13.0 s vs. 19.7 s for round-down and round-up, respectively) than in sixth graders (7.5 s vs. 12.7 s) and was the smallest in adults (4.3 s vs. 6.8 s), as shown by the significant age \( \times \) strategy interaction, \( F_1(2, 138) = 3.45, MSe = 63.47; F_2(2, 136) = 158.55, MSe = 0.99 \). As in choice conditions, results on mean percent deviations did not compromise these findings on latencies. The only effect that came out significant in both participant- and problem-based analyses was that of age, showing that older participants provided better estimates: mean percent deviations were 1.97, 1.70, and 1.59 in fourth graders, sixth graders, and adults, respectively, \( F_1(2, 138) = 3.34, MSe = 2.16; F_2(2, 136) = 16.71, MSe = 0.32 \). Finally, the strategy \( \times \) unit size interaction was significant only in participant-based analyses, \( F_1(1, 138) = 317.45, MSe = 0.29; F_2 < 1 \). These cross-over patterns are evidence that participants complied with the requested strategy instructions. Mean percent deviations are expected to be smaller with round-down than with round-up on small-unit problems (and the reverse on large-unit problems). To illustrate, on a small-unit problem like 263 + 471, correct execution of round-down and round-up produces 0.5% and 2.1% deviations, respectively, while corresponding percent deviations are 1.8% and 0.3% for each strategy, respectively, on a large-unit problem like 284 + 657. No other effects were significant in participant- or problem-based analyses of strategy performance under either choice or no-choice condition. In summary, analyses of mean solution latencies and percent deviations revealed that round-down strategy yielded shorter solution times and better estimates than round-up strategy, and even more so in younger children.

Effects of problem size on strategy execution

Problem-size effects are one of the most robust effects in arithmetic (see Ashcraft, 1995, for a review) and were found in one previous study of computational estimation (LeFevre et al., 1993). Problem-based correlations between two measures of problem size (i.e., correct sum and sum of hundred and decade digits), strategy solution times, and percent deviations in no-choice were calculated for each age group. Results indicated that strategy solution times and percent deviations under no-choice conditions decreased with increasing problem size in each age group. Sum of hundred and decade digits was the only variable that correlated significantly with no-choice strategy speed in adults \( (rs = .89 \) and .87 for round-down and -up strategies,
respectively), sixth graders ($rs = .89$ and $.78$), and fourth graders ($rs = .88$ and $.80$). Corresponding correlations between sum of hundred and unit digits and mean percent deviations for round-down and -up strategies in no-choice were $.36$ and $.35$ in adults, $.42$ and $.37$ in sixth graders, and $.42$ and $.53$ in fourth graders. These findings indicate that computational estimation strategy execution was influenced by problem size in all age groups.

**Effects of having the choice on strategy execution**

As different participants were tested under choice and no-choice conditions and in each no-choice condition, solution times and percent deviations for each round-up and -down strategy were compared via independent t tests. As it can be seen in Figs. 2 and 3, both round-up and -down strategies yielded equally good estimates under choice and no-choice conditions, $ts(46) < 1.00$. However, round-down strategy was executed more quickly under no-choice than under choice conditions in all age groups, $ts(46) > 2.85$, and round-up strategy was executed equally fast under choice and no-choice conditions, $ts(46) < 1.32$. ³

**General discussion**

This experiment documented children age-related differences in computational estimation strategy use and execution. It replicated previous findings regarding children’s computational estimation skills and documented age-related changes in strategic aspects unknown before. Two sets of interesting findings, each concerning strategy use and execution, respectively, have implications for further understanding age-related changes in strategic behaviors, in general, and computational estimation, in particular. We discuss each of these implications in turn.

The first interesting set of findings concerns strategy use. All participants favored round-down over round-up and use of round-down was influenced by problem, strategy, and participant characteristics. First, participants of all age groups were influenced by unit size (a problem feature), with round-down being used more often on small-unit problems and round-up more often on large-unit problems. Furthermore, decade size accounted for substantially less variance in fourth graders’ strategy use than the unit size variable did in the strategy choices of adults’ and sixth graders’. In summary, younger children were less systematic in their strategy use than older children and adults, who did not differ.

Another original finding in this study concerns age-related differences in the influence of relative strategy performance on strategy use. Both adults’

³ The same outcomes were observed when choice/no-choice comparisons were restricted to non-mixed problems.
and sixth graders’ use of round-down significantly correlated with difference in mean percent deviations. Nine-year-olds’ strategy choices were not correlated with this parameter. These data suggest that older participants’ strategy choices were calibrated to relative strategy performance. However, note that future studies should collect data that directly address this issue, as only problem-based correlations could be calculated in the present study. To really know whether participants’ strategy choices are fine-tuned to relative strategy performance, the same participant should be tested under the choice and no-choice conditions. Note though that previous studies using the choice/no-choice variable as a within-participant variable found such strategy choice calibrations (e.g., Lemaire & Lecacheur, 2001; Siegler & Lemaire, 1997) and age-related increase in these calibrations (Lemaire & Lecacheur, 2002).

From a developmental perspective, computational models of strategy choices, such as ASCM or SCAD (Shrager & Siegler, 1998), suggest that age-related changes in correlations between strategy choices and strategy performance are the result of increased participants’ expertise in arithmetic. Nine-year-olds know arithmetic facts less well than older participants (e.g., Ashcraft & Fierman, 1982; Bisanz, Frederick, & Dunn, 1995; Geary & Brown, 1991; Geary, Christine, Liu, & Siegler, 1996; Lemaire et al., 1994). This relative lack of arithmetic expertise may have consequences on different processes involved in computational estimation, such as calculation and rounding processes. It has often been observed that children are less at ease than adults with computational tasks (e.g., Case & Sowder, 1990; LeFevre et al., 1993; Reys et al., 1982). Similarly, adults and children of varying mathematical proficiencies have been found to differ in computational estimation skills (Dowker et al., 1996, 1997). Although none of our children had troubles understanding the goal of the task and perfectly understood the differences between calculation and computational estimation tasks, they may have had more difficulties than older participants and, hence, showed less strategy adaptiveness.

The final set of interesting findings in this study concerns strategy execution. Contrary to previous works on computational estimation, children’s strategy execution was not biased here by selective use of strategies across problems and/or participants. No-choice performance revealed that participants were faster and found better estimates while using round-down. It also revealed that strategy solution times and percent deviations were influenced by age and problem characteristics. Detailed task analyses of cognitive processes involved in each strategy account for this phenomenon of relative strategy performance. Although both strategies involved the same cognitive operations (i.e., encoding operands, rounding operands, calculating sums of rounded operands, and saying these sums aloud), task demands were different for each cognitive operation when involved in each strategy. More precisely, encoding could be easier when involved in round-down than in round-up as round-down could be executed accurately without encoding unit
digits. Moreover, rounding processes were probably easier to execute in round-down than in round-up for two reasons. Round-down did not require calculating differences between unit digits and the closest larger decades; this calculation was required in round-up. In addition, round-up processes had to be executed with decade digits stored in working memory whereas working memory was much less involved in executing rounding-down processes, as the manipulated decade digits were displayed on the computer screen while trying to find estimates. Finally, easier calculation for round-down may have also contributed to its yielding better performance. Round-down was always involved in adding smaller numbers than round-up, a problem feature that is known to play a crucial role in both adults’ and children’s arithmetic (see Ashcraft, 1995; Geary, 1996, for recent reviews). Together, these sources of strategy speed differences cumulated to lead younger participants to be slower than older participants while executing round-up strategy. This is a basic age \times difficulty effect, presumably resulting from younger participants’ triggering and executing each cognitive operation within strategies more slowly.

Future studies may help understand a puzzling pattern of results in the present experiment. That is, contrary to previous research in other cognitive activities and to what formal models of strategy choices such as Siegler and collaborators’ ASCM/SCAD model (Shrager & Siegler, 1998) would predict, none of our participants obtained better performance in choice than in no-choice conditions. We expected such patterns because people can choose strategies that work best on each problem in choice condition. In past research, choice conditions enabled both adults and children to obtain better performances, as their adaptive strategy choices enabled them to optimize their performance and to compensate for the extra-time-consuming step of deciding for each item, which strategy to choose. Here, performance was not better in choice conditions than in no-choice conditions. Two potential reasons may be tested in future studies. First, computational estimation is not a well-practiced task. Choice benefits may hold only when participants choose among strategies they have often practiced pre-experimentally, like exact calculation tasks (Siegler & Lemaire, 1997) or spelling (Lemaire & Lecacheur, 2002). Such a possibility was observed in Lemaire and Lecacheur’s (2001) study in which adults used two new strategies and in which no-choice solution times were shorter than choice solution times. A microgenetic design (Siegler & Crowley, 1991) testing the same participants a number of times may help determine whether sufficient practice with estimation tasks and strategies leads to better performance in choice conditions. This would suggest that practice helps people making fast choice in addition to increasing speed of strategy execution.

The second reason that remains to be tested before concluding that choice conditions do not yield better performance than no-choice conditions in computational estimation tasks concerns the strategies that were used here. Recall that participants could use only two strategies, round-down
and round-up, and that there were three types of problems (i.e., problems for which unit digits of both operands were smaller than 5; problems for which unit digits of both operands were larger than 5; and problems for which unit digits of one operand were smaller than 5 and unit digits of the other operand were larger than 5). These restrictions were adopted here so that children would not pre-experimentally or early in the experiment adopt a meta-strategy by which they would use round-up with problems including unit digits of both operands larger than 5 and round-down with problems including unit digits of both operands smaller than 5. The so-called mixed problems (i.e., those including unit digits of one operand smaller than 5 and unit digits of the other operand larger than 5) were used here not only to control for such potential artifacts but also to confront participants with a set of representative three-digit operand problems. Unfortunately, this may have had the consequences that, as they said either during or after the experience, participants would prefer using both strategies (i.e., rounding one operand down and the other up) on some problems. Such a mixed strategy could be used for one-third of the problems in our experiment. Had participants been given this opportunity or been presented with no mixed problems they may have obtained faster solution times in choice conditions. They may also have selected strategies more systematically than here on the basis of relative strategy speed and/or benefits.

Future studies of age-related changes in computational estimation using the choice/no-choice method and giving participants the opportunity to use three strategies (rounding-down, rounding-up, and mixed strategies) should (a) test the generality of the present findings to cases where more than two strategies are available, (b) tell how children would execute (in the no-choice condition) and use (in the choice condition) the mixed strategy in comparison to other rounding strategies, and (c) determine whether, like in other cognitive domains, having the choices between several strategies yields better performance under choice than under no-choice conditions and whether children are more and more finely tuned to relative strategy execution as they gain expertise. More specifically, such data would help determine whether performance differences between choice/no-choice conditions hold for all available strategies and all types of problems, how participants would use the two variants of the mixed strategy (i.e., rounding down the first operand and round up the second operand, or the reverse), whether participants of varying ages would systematically round the first operand down and the second operand up on problems with the unit digit of the first operand smaller than 5 and that of the second operand larger than 5 (e.g., 123 + 767) and do the reverse on problems like 127 + 763, and whether there are age-/skill-related differences in using and executing these variants.

In addition to addressing specific issues (such as differences in strategy execution under choice and no-choice conditions), future research may have the goal of testing the generality of our present findings within and outside...
the domain of computational estimation. Within estimation, such studies may investigate age-related changes in computational estimation performance as a function of different contexts of estimation (e.g., speed vs. accuracy pressures, ecological vs. laboratory-based computational estimation tasks) or individual differences (e.g., high-/low-arithmetic skill participants, younger vs. older adults).

Finally, the generality of the present findings on strategic changes may be tested with the same research approach as the one adopted here to further our understanding of strategic changes in a number of cognitive domains. In cognitive areas as diverse as arithmetic, tic-tac-toe, serial recall, spelling, and moral or scientific reasoning, previous research has shown that children of varying ages know and use several strategies (e.g., Bisanz & LeFevre, 1990; Bjorklund & Rosenblum, 2001; Colby, Kohlberg, Gibbs, & Lieberman, 1983; Crowley & Siegler, 1993; Geary & Burlingham-Dubree, 1989; Goldman, Mertz, & Pelegrino, 1989; Hock, Park, & Bjorklund, 1998; McGilly & Siegler, 1990; Kuhn & Phelps, 1982; Marsh, Friedman, Welch, & Desberg, 1980; see Siegler, 1996, for an overview). In all these domains, age-related differences in strategic changes can be fruitfully investigated via the choice/no-choice method used here.

References


