Children's strategies in complex arithmetic

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Abstract
Strategies used to solve two-digit addition problems (e.g., 27 + 48, Experiment 1) and two-digit subtraction problems (e.g., 73 – 59, Experiment 2) were investigated in adults and in children from Grades 3, 5, and 7. Participants were tested in choice and no-choice conditions. Results showed that (a) participants used the full decomposition strategy more often than the partial decomposition strategy to solve addition problems but used both strategies equally often to solve subtraction problems; (b) strategy use and execution were influenced by participants' age, problem features, relative strategy performance, and whether the problems were displayed horizontally or vertically; and (c) age-related changes in complex arithmetic concern relative strategy use and execution as well as the relative influences of problem characteristics, strategy characteristics, and problem presentation on strategy choices and strategy performance. Implications of these findings for understanding age-related changes in strategic aspects of complex arithmetic performance are discussed.

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Introduction

Research in arithmetic aims at understanding how people accomplish arithmetic problem solving tasks and how arithmetic skills change with participants' age. Examining determiners of participants' performance has helped to build models of arithmetic processing. The current study aimed at further understanding how children and adults solve complex, two-digit arithmetic problems. To achieve this end, we investigated strategic aspects of their performance. Before outlining the logic of the current work, we briefly review previous findings on complex arithmetic problem solving.

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Most researchers agree that when participants solve complex arithmetic problems, they need to encode digits, find arithmetic facts in long-term memory (LTM) or count, temporarily hold partial results, and add multiple digits. Above and beyond these elementary processes, previous research on complex addition and subtraction problems showed several important findings that guided the current work. First, both adults and children use several strategies. A strategy can be defined as “a procedure or a set of procedures for achieving a higher level goal or task” (Lemaire & Reder, 1999, p. 365). To solve multidigit addition and subtraction problems, both children and adults use several strategies such as direct retrieval of the correct solution in LTM, several different transformation strategies (e.g., to solve 45 + 39, they can do 45 + 40 – 1, 40 + 40 + 5 – 1, or 50 + 34), or several decomposition strategies (e.g., to solve 45 + 39, they can do 40 + 30 + 5 + 9, 45 + 30 + 9, 40 + 39 + 5, or 5 + 9 + 30 + 40). Such strategy variability has been observed for both addition and subtraction problems in children and adults of all ages (e.g., Arnaud, Lemaire, Allen, & Michel, 2008; Beishuizen, 1993; Beishuizen, Van Putten, & Van Mulken, 1997; Lemaire & Lecacheur, 2001; Lucangeli, Tressoldi, Bendotti, Bonanomi, & Siegel, 2003). Two of the complex arithmetic strategies have been the focus of investigation in several past studies. These are the full and partial decomposition strategies that both children (as young as 7 years) and adults use. In the full decomposition strategy, both addends are split into tens and units, which are added separately and then are combined again (e.g., 53 + 46: 50 + 40 = 90, 3 + 6 = 9, 90 + 9 = 99). In the partial decomposition strategy, only the second addend is split up and the tens and units are added onto the unsplit first addend (e.g., 34 + 63: 34 + 60 = 94, 94 + 3 = 97). The current study also focused on these two strategies.

Complex arithmetic strategies have been found to be used with different proportions in different age groups. For example, Lucangeli and colleagues (2003) found that third, fourth, and fifth graders used different strategies to solve multidigit addition, subtraction, multiplication, and division problems. Regarding the two strategies that are the focus of the current work, third graders used the full decomposition strategy less often than fourth and fifth graders (14, 31, and 28%, respectively, for addition problems and 2, 8, and 13%, respectively, for subtraction problems) and the three age groups used the partial decomposition strategy equally often (3% for addition problems and 9% for subtraction problems). The full decomposition strategy was favored by second graders, whereas third graders used both the full and partial decomposition strategies equally often (Beishuizen et al., 1997; Blöte, Van der Burg, & Klein, 2001; Fuson, 1990).

The second robust empirical finding relevant to the current project concerns the role of problem characteristics in participants’ performance and strategy use such as size of the operands and whether problems involve carryover or not. For example, Green, Lemaire, and Dufau (2007) found that adults used the unit strategy (i.e., adding units first, then decades, and then hundreds) more often with carry problems than with no-carry problems while solving three-digit addition problems. Imbo, Vandierendonck, and De Rammelaere (2007) found that participants’ performance decreased as the number and value of carries increased in multidigit addition problem solving.

The third empirical finding that is relevant to the current project is that arithmetic processes and performance are influenced by a number of different situational constraints. For example, participants change their strategies and have different levels of performance when they are tested under varying time–accuracy pressure conditions (e.g., Campbell & Austin, 2002), when they are or not asked verbal protocols regarding how they solved problems (Kirk & Ashcraft, 2001; LeFevre, DeStefano, Penner-Wilger, & Daley, 2006), and when problems are presented under different formats (e.g., Campbell & Fugelsang, 2001; LeFevre, Lei, Smith-Chant, & Mullins, 2001; Noël, Fias, & Brysbaert, 1997). For example, LeFevre and colleagues (2001) observed a smaller problem size effect with auditory stimuli than with Arabic stimuli while participants were solving simple multiplication problems. The role of these situational constraints has been much less investigated in children, although existing data suggest that such constraints also affect their arithmetic strategies and performance. For example, Lucangeli and colleagues (2003) found that 8- to 10-year-olds did not use the same strategies to solve mental versus written addition and subtraction problems.

Previous research on strategic aspects of children’s complex arithmetic has been limited in several respects. First, no studies have examined age-related changes in how often children use complex arithmetic strategies.
arithmetic strategies when strategies are assessed on a trial-by-trial basis. Therefore, we do not know whether the frequency with which children use different strategies changes with development for complex arithmetic. To determine how often each strategy is used by individuals as well as age-related differences in distributions of strategies, it is necessary to assess strategy use on each trial.

Second, because strategies are used unequally often by participants of different groups and on different types of problems, strategy use and execution are confounded. This makes it impossible to compare strategy use and execution across different groups of children and to know whether relative strategy performance stems from true differences in strategy speed and accuracy or from strategy selection (over items or participants) artifacts. To examine relative strategy efficiency and how it changes with age, it is important that the frequencies of strategy use and the problems on which each strategy is used are the same across age groups. We did this in the current study.

Third, no studies have investigated the determiners of strategy choices, so that we do not know whether, like in simple arithmetic, complex arithmetic strategies are influenced by problem characteristics (e.g., size of operands) and/or strategy characteristics (e.g., strategy speed). We also do not know how effects of problem and strategy characteristics on strategy use change with children’s age. Here we tested whether the use of complex arithmetic strategies is influenced by problem and/or strategy characteristics and how the effects of these factors change with children’s age.

Fourth, although previous studies have shown that problem features affect children’s strategy use and their arithmetic performance, one problem feature has received much less attention—carryover. No studies have investigated systematically the role of carries on children’s strategy use, how children execute carry processes, and how carry processing changes with age. Because carry processing puts a large burden on working memory resources (see DeStefano & LeFevre, 2004, and LeFevre, DeStefano, Coleman, & Shanahan, 2005, for detailed discussion and review of the role of working memory in arithmetic performance), and because children’s working memory resources increase with age (see Hitch, 2006, for a review), we tested the hypothesis that carry processing becomes more efficient as children age. More specifically, we predicted that the differences in performance to solve carry and no-carry problems decrease as children’s age increases.

Finally, very few studies have investigated how task constraints affect children’s arithmetic performance and processes. Therefore, we do not know how such situational constraints influence children’s arithmetic and how these effects of task constraints change with children’s age. For example, no studies have compared children’s performance in conditions where problems are presented vertically versus horizontally while each participant used both strategies equally often and on the same problems. We compared strategy use and strategy execution for horizontally and vertically presented problems. Such a comparison was important not only to understand the role of task constraints in children’s arithmetic but also because most studies of children’s arithmetic have tested horizontally presented problems. Thus, the current study offered the opportunity to generalize conclusions from these studies to vertically presented problems.

Overview of the current study

The major goal of this project was to contribute to our further understanding of age-related changes in children’s complex arithmetic. We adopted a perspective in which we studied performance by controlling the strategies used by children and adults to solve multidigit arithmetic problems.

One group of adults and two groups of children solved two-digit addition problems (Experiment 1) or subtraction problems (Experiment 2). Based on previous studies (e.g., Beishuizen, 1993; Beishuizen et al., 1997; Lucangeli et al., 2003), third and fifth graders solved addition problems (Experiment 1) and fifth and seventh graders solved subtraction problems (Experiment 2). All participants were tested in both the choice and no-choice conditions (Siegler & Lemaire, 1997). In the choice condition, participants were allowed to choose between the full and partial decomposition strategies on each problem. In the no-choice condition, participants solved all problems first with one of the two strategies and then with the other strategy.

The data collected in this study enabled us to address the following issues regarding age-related changes in strategic dimensions of complex arithmetic performance: Do participants have preferences among full and partial decomposition strategies? Are strategy use and execution influenced by
participants’ age, problem features (carryover and size of operands), strategy characteristics (relative strategy performance), and/or task constraints (presentation format)? How do effects of problem type, strategies, and task constraints change with children’s age? How does processing carries change with children’s age?

We hypothesized that participants’ performance in the no-choice conditions would reflect their preferred strategy in the choice condition. In particular, we expected that the strategy most often chosen by an individual would also be the faster and more accurate one in the no-choice condition. Furthermore, we expected that participants would perform better while solving no-carry problems than while solving carry problems and would use full decomposition more on carry problems than on carry problems because it is harder to execute on the latter. It was hard to make specific predictions regarding effects of presentation format because no previous studies had compared problems presented both horizontally and vertically. Finally, age-related differences in strategy use and execution predict different strategy preferences in each age group, increased strategy performance in older participants, and age-related differential influences of problem features (i.e., decreased differences in performance between carry and no-carry problems) and task constraints (i.e., different effects of presentation format as a function of children’s age).

**Experiment 1: Addition problem solving**

**Method**

**Participants**

A total of 90 individuals—30 adults (20 women and 10 men), 30 fifth graders (16 girls and 14 boys), and 30 third graders (16 girls and 14 boys)—participated in this experiment. The adults were undergraduate college students who received course credit for participating in the experiment. Their mean age was 21 years 4 months (range = 18 years 1 month to 22 years 5 months). The fifth graders had a mean age of 10 years 6 months (range = 9 years 9 months to 11 years 1 month), and the third graders had a mean age of 7 years 6 months (range = 6 years 7 months to 8 years 1 month). Children were from a French upper class urban public school.

**Stimuli**

The stimuli were 72 addition problems presented in a standard form (i.e., \(a + b\)) in which \(a\) and \(b\) were two-digit numbers, the sum of which was smaller than 99 (mean correct sum = 73, SD = 17.3, range = 35–98). The set of 72 problems was divided into three subsets of 24 problems each. The three subsets of problems were matched on their correct sums (means were 73 for all three subsets), standard deviations of correct sums (standard deviations were 16.9, 17.7, and 18.1 for the first, second, and third subsets, respectively), minimum correct sums (37, 37, and 35, respectively), and maximum correct sums (98 for all three subsets). Half of the problems had a carry in the unit position (e.g., 18 + 24), and the other half had no-carry (e.g., 12 + 36).

Based on previous research in arithmetic (e.g., Campbell, 2005; Geary, 1994), problems were selected with the following constraints. First, the larger of the two operands was in the left position (e.g., 67 + 26) in half of the problems and was in the right position (e.g., 18 + 73) in the other problems. Second, the operand with the smallest unit digits was in the left position (e.g., 12 + 36) in half of the problems and was in the right position (e.g., 23 + 41) in the other problems. Third, no operand had 0 or 5 as a unit digit. Fourth, digits were not repeated in the same unit or decade positions across operands (e.g., 64 + 24). Fifth, no digits were repeated within operands (e.g., 66 + 31). Sixth, no reverse orders of operands were used (e.g., if 23 + 41 was tested, 41 + 23 was not used), carry and no-carry problems were matched on the size of the correct sums (mean correct sums were 73).

**Procedure and materials**

Participants were tested individually in one session that lasted approximately 45 to 60 min. They were told that they would see two-digit addition problems on the computer screen and were asked to solve each problem. They were told how many blocks, trials within each block, and practice trials they
would complete. They were told to calculate out loud and to say the answer aloud as quickly as possible. Then each strategy was explained, illustrated, and practiced on three carry and three no-carry problems. After this initial practice period, no participants had difficulties with either strategy. Instructions stressed speed and precision equally.

Each participant solved problems under three conditions. In the choice condition, participants could use the full decomposition strategy or the partial decomposition strategy to solve the problem. The two strategies were described as follows:

To use the full decomposition strategy, you first add the tens. Then you add the units. Finally, you add together the two results. For example, for 24 + 53, you first do 20 + 50 = 70. Then you do 4 + 3 = 7. Finally, you do 70 + 7 = 77 and give 77 as a result. For 28 + 34, you do 20 + 30 = 50, 8 + 4 = 12, 50 + 12 = 62. To use the partial decomposition strategy, you first add the tens of the second number to the first number. Then you add this result to the unit of the second number. For example, for 24 + 53, you first do 24 + 50 = 74. Then you do 74 + 3 = 77 and give 77 as a result. For 28 + 34, you do 28 + 30 = 58, 58 + 4 = 62.

In the full decomposition–no-choice condition, participants were required to use the full decomposition strategy to generate an answer to each problem. In the partial decomposition–no-choice condition, they needed to use the partial decomposition strategy to solve all problems. Each subset of problems was seen by an equal number of participants under each of the three conditions. The subsets of problems were equivalent in the speed and accuracy they elicited in the no-choice conditions and in the percentage of choices of partial decomposition they elicited in the choice condition, $F_s < 1.10$.

Problems were presented in 72-point Times New Roman black font on a white background in the center of an IBM-compatible computer monitor. Participants sat 60 cm from the 800 × 600-pixel computer screen. Problems were presented horizontally. Each digit and symbol occupied a 4 × 1.5-cm (height × width) predefined zone on the computer screen, with each one corresponding to 3.8° (height) and 1.5° (width) visual angles. Zones were 1 cm apart from each other. E-Prime software controlled the stimulus display and collected response times (RTs). Each trial began with the 750-ms presentation of a fixation point (•) in the center of the computer screen. Then the problems were displayed horizontally in the center of the screen. The timing of each trial began when the problem appeared on the screen and ended when the experimenter pressed a button, with the latter event occurring as soon as possible after the participant’s response. On each trial, the experimenter recorded participants’ responses. Moreover, in choice conditions, participants needed to say which strategy was used on each trial. Because participants were asked to calculate out loud, verbal reports and strategies inferred by the experimenter from participants’ solving the problem out loud agreed on all trials. Also, verbal reports indicated that participants used the strategy they were asked to use on all no-choice trials.

Participants were randomly assigned to one of two presentation orders: choice/partial decomposition–no-choice/full decomposition–no-choice or choice/full decomposition–no-choice/partial decomposition–no-choice. The choice condition was always presented first so that choices would not be influenced by recency effects from having just used a given strategy on 24 consecutive trials. Participants were given short rest periods following completion of the first and second subsets of problems.

Results

Results are presented in two main parts. The first analyzes age-related differences in strategy use, and the second examines age-related differences in strategy execution. In all results, unless otherwise noted, differences are significant to at least $p < .05$.

Age-related differences in strategy use

This section analyzes strategy use in the choice condition. First, we analyze strategy variability. More precisely, we ask whether participants used only one strategy or both strategies, whether both strategies were used equally often (if both strategies were used), and whether strategy use was influenced by participants’ age and carry.
Variability in strategy use. The first question concerning strategy use was whether participants solved addition problems with only one strategy or both strategies. In other words, was there any within-participant variability in strategy use? One way to answer this question is to tally the number of participants using full decomposition on each of the three different ranges of problems so as to know how many participants used only one strategy and how many used full decomposition (or partial decomposition) most often. Strategy variability can be seen in Table 1. No participants used full decomposition on either 0 or 100% of problems, and no problems were solved with full decomposition by either all or none of the participants.

Then we analyzed mean percentages of the use of full decomposition with analyses of variance (ANOVAs) involving a 3 (Group: adults, fifth graders, or third graders) × 2 (Carry: present or absent) design with age as the only between-participant factor.

As can be seen from Fig. 1, participants of all age groups used full decomposition most often (59%). Participants used the full decomposition strategy more often on problems with no-carry than on problems with carry (62 vs. 56%), F(1, 87) = 4.61, MSe = 412.19. The main effect of age was not significant, F < 1, with adults using full decomposition on 56% of problems and fifth and third graders using it on 62 and 58% of problems, respectively. Moreover, the Age × Carry interaction, F(2, 87) = 4.88, MSe = 412.9, was significant. Both third and fifth graders used full decomposition equally often while solving problems with and with no-carry, Fs < 1. In contrast, adults used it more often while solving problems with no-carry than while solving problems with carry (64 vs. 48%), F(1, 29) = 10.06, MSe = 414.30.

Determiners of strategy use. Previous works on strategy use showed that both children and adults adjust the strategies they use to two types of parameters, namely, problem and strategy characteristics (for a review, see Siegler, 1996). Problem characteristics refer to a variety of features such as problem size, whether problems involve carries or not, and side of the larger operand. Strategy characteristics refer to relative strategy performance with a given strategy (e.g., Lemaire & Lecacheur, 2002a; Lemaire & Lecacheur, 2002b). The goals of these analyses were to determine (a) whether strategy use in complex arithmetic problem solving is also influenced by both problem and strategy characteristics, (b) whether these influences vary with age, (c) which of the problem and strategy characteristics is the best predictor of strategy use in complex arithmetic problem solving, and (d) whether this predictor changes with age.

First, to know whether participants adjusted their strategy use to problem and/or strategy characteristics, we ran a series of correlation analyses. Problem-based correlations in each age group were calculated between mean percentage use of full decomposition on each problem and two types of variables. One set of variables characterized problem features, and the other set characterized relative strategy performance. Problem features included (a) correct sums (e.g., 48 for 12 + 36), (b) side of the largest operand (coded 0 for left position and 1 for right position), and (c) carry problems (coded 1).
versus no-carry problems (coded 0). Relative strategy performance included (a) relative strategy precision (i.e., how accurate performance is with each strategy) and (b) relative strategy speed (i.e., how fast participants are with strategies). Relative strategy precision was calculated for each problem in two steps. First, mean percentage errors were calculated for each strategy and each problem. Second, differences between these mean percentage errors were calculated. Thus, the formula for relative strategy precision was as follows: full decomposition/no-choice mean percentage errors – partial decomposition/no-choice mean percentage errors. Similarly, relative strategy speed was calculated with the following formula: RTs for full decomposition/no-choice – RTs for partial decomposition/no-choice. Significant correlations between mean percentage use of full decomposition and problem or strategy characteristics were expected to be the signatures of participants’ strategy use being influenced by these two types of variables. The correlation matrix is presented in Table 2.

The influence of problem and strategy characteristics on strategy use was not the same in each age group. None of the problem characteristics correlated with mean percentage use of full decomposition in either group of children ($r_s < .12$). Adults’ strategy use correlated with one problem characteristic, carry ($r = -.32$), such that, consistent with ANOVA results, adults used full decomposition more often when problems had no-carry. Strategy characteristics and strategy use correlated significantly in all age groups. Fifth and third graders’ use of full decomposition correlated significantly with both relative strategy precision ($r_s = -.48$ and $-.52$, respectively) and relative strategy speed ($r_s = -.55$ and $-.26$, respectively). These show that children used full decomposition more often when it yielded correct and fast answers. Adults’ use of full decomposition correlated significantly with relative strategy precision ($r = -.49$) but not with relative strategy speed ($r = -.08$).

These results were confirmed in stepwise regression analyses conducted separately in each age group to determine which of the two types of variables—problem or strategy characteristics—best predicts the mean percentage use of full decomposition on each problem. Results showed that relative strategy precision was the best predictor in adults ($R^2 = .24$). The carry variable accounted for 11% additional unique variance in adults’ strategy use. The best single predictor of full decomposition was relative strategy precision in fifth graders ($R^2 = .28$) and relative strategy speed ($R^2 = .30$) in third graders. Although strategy use and relative strategy speed correlated significantly in fifth graders, it did not do so in the regression analyses. The reason for this was that all of the variance captured by this variable was also captured by relative strategy precision, and the latter variable was more highly correlated with the percentage use of full decomposition on each problem. Similarly, strategy use correlated with relative strategy precision in third graders but did not do so in the regression analyses.
Again, this is because all of the variance captured by relative precision was also captured by relative strategy speed, and the latter variable correlated more highly with the percentage use of full decomposition on each problem.

**Age-related differences in strategy execution**

This section analyzes strategy execution in both the choice and no-choice conditions so as to examine effects of age and carry on strategy execution and to determine how the rate of processing carry changes with age for each strategy.

**Effects of age, strategy, and carry on performance.** Mean solution latencies and percentage errors in both the choice and no-choice condition were analyzed with ANOVAs. ANOVAs were run with 3 (Group: adults, fifth graders, or third graders) × 2 (Strategy: full decomposition or partial decomposition) × 2 (Carry: present or absent) designs with repeated measures on the last two factors. In both Experiments 1 and 2, the same effects came out significant in the choice and no-choice conditions. Therefore, we report only analyses of the no-choice condition performance because they were not contaminated by strategy selection. In both experiments, analyses of solution times were also conducted on standardized solution times to control for artifactual interactions involving the age factor due to general speeding. Because no interactions involving the age factor were artifactual in the current experiments, we report statistics for raw estimation times.

As can be seen in Table 3, adults were faster (5.9 s) than fifth graders (10.3 s), who in turn were faster than third graders (14.5 s), F(2, 87) = 23.07, MSe = 96.15. Moreover, the carry effect, F(1, 87) = 186.44, MSe = 3.94, showed that participants were faster while solving problems with no-carry (8.8 s) than while solving problems with carry (11.7 s). The main effect of strategy was not significant, F < 1, with participants being equally fast with the full decomposition strategy (10.9 s) and the partial decomposition strategy (11.1 s). The Age × Carry interaction, F(2, 87) = 7.41, MSe = 3.94, showed that size of carry effects decreased with increasing age, such that they were larger in third graders (3.7 s), F(1, 29) = 129.76, MSe = 2.21, than in fifth graders (3.1 s), F(1, 29) = 49.44, MSe = 8.34, or in adults.
Table 3
Mean solution times and percentage errors for full and partial decomposition strategies in each group of participants and for each problem type (Experiment 1, no-choice conditions)

<table>
<thead>
<tr>
<th></th>
<th>Solution times</th>
<th>Percentage errors</th>
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<tbody>
<tr>
<td></td>
<td>Full decomposition</td>
<td>Partial decomposition</td>
</tr>
<tr>
<td></td>
<td>No-carry Carry Mean</td>
<td>No-carry Carry Mean</td>
</tr>
<tr>
<td>Adults</td>
<td>4.7 6.7 5.7</td>
<td>5.4 6.9 6.1</td>
</tr>
<tr>
<td>Fifth graders</td>
<td>8.6 11.7 10.2</td>
<td>8.9 11.9 10.4</td>
</tr>
<tr>
<td>Third graders</td>
<td>12.6 16.4 14.5</td>
<td>12.7 16.3 14.5</td>
</tr>
</tbody>
</table>

Note. Solution times are in seconds (s).

(1.8 s), \( F(1, 29) = 74.07, \text{MSe} = 1.23 \). Analyses of errors showed that adults were most accurate (4.6%), followed by fifth graders (5.2%) and then third graders (7.1%), \( F(2, 87) = 2.67, \text{MSe} = 75.95 \). Participants were more accurate when they used the full decomposition strategy than when they used the partial decomposition strategy (5.0 vs. 6.3%), \( F(1, 87) = 4.98, \text{MSe} = 5.21 \), and they were more accurate when they solved no-carry problems than when they solved carry problems (3.0 vs. 8.3%), \( F(1, 87) = 64.63, \text{MSe} = 39.13 \). Finally, the Strategy \( \times \) Carry interaction was marginally significant, \( F(1, 87) = 3.52, p = .06, \text{MSe} = 14.92 \). Carry effects tended to be larger for the partial decomposition strategy (6.1%) than for the full decomposition strategy (4.6%). No other effects were significant on participants' performance.

**Age-related changes in the speed of carry processing.** To analyze how the speed of carry processing changes with age, we conducted componential analyses. All of these analyses were based on correct RT trials in the no-choice conditions. Following Geary and collaborators (e.g., Geary, Frensch, & Wiley, 1993), process models were fitted to individual RT data using regression techniques. The carry variable was used to model solution times for each strategy in each age group. The raw regression weight (which is termed the component score) for the carry variable provides an estimate for the speed of carry processing. The intercept term provides a combined estimate for the duration of all other processes (number encoding, answer production, and columnar retrieval). To illustrate, the component score for the carry variable for our adult participants was on average 2.0 s when they used the full decomposition strategy and 1.6 s when they used the partial decomposition strategy. These values indicate that adults needed 2.0 s to process carry while using the full decomposition strategy and needed 1.6 s while using the partial decomposition strategy.

Mean component scores across group and strategy are displayed in Table 4. Effects of age on intercept, \( F(2, 87) = 22.73, \text{MSe} = 40.33 \), and on slope, \( F(2, 87) = 6.34, \text{MSe} = 7.55 \), were significant. For both slope and intercept, there were no significant effects of strategy, \( F < 2.37 \), or Age \( \times \) Strategy interaction, \( F < 1 \). These analyses revealed that as participants grow older, they become more proficient at processing carries and at other arithmetic processes (number encoding, answer production, and columnar retrieval) while they use the partial decomposition or full decomposition strategy.

**Experiment 2: Subtraction problem solving**

**Method**

**Participants**

A total of 56 individuals—20 adults (15 women and 5 men), 18 seventh graders (10 girls and 8 boys), and 18 fifth graders (10 girls and 8 boys)—participated in the study. The adults were undergraduate college students who received course credit for participating in the experiment. Their mean age was 23 years 5 months (range = 18 years 0 months to 30 years 1 month). The seventh graders had a mean age of 12 years 3 months (range = 12 years 0 months to 13 years 1 month), and the fifth graders had a mean age of 9 years 9 months (range = 9 years 1 month to 10 years 1 month). Children were from a French upper class urban public school.
Stimuli
The stimuli were 96 two-digit subtraction problems, the correct differences of which ranged from 11 to 62. The set of 96 problems was divided into three subsets of 32 problems each. The three subsets of problems were matched on their correct differences (means were 29 for all three subsets), standard deviations of correct differences (standard deviations were 14.9, 14.1, and 14.7 for the first, second, and third subsets, respectively), minimum correct differences (11, 12, and 12, respectively), and maximum correct differences (62, 61, and 61, respectively). Half of the problems had a borrow (i.e., difference in the unit digits was smaller than 0) in the unit position (e.g., 31 – 19), and the other half had no-borrow (e.g., 38 – 26).

The same set of potential confounds as in Experiment 1 was controlled. Moreover, no correct differences in either units or decades were equal to 0 (e.g., 36 – 16), and no correct differences were equal to the second operand (e.g., 24 – 12).

Procedure and materials
The procedure and materials were exactly the same as in Experiment 1, with two exceptions. First, the two full and partial decomposition strategies involved different procedures for subtraction problems. The strategies were described as follows:

To use the full decomposition strategy, you first subtract the tens. You then subtract the units. Finally, you add together the two results. For example, for 54 – 23, you first do 50 – 20 = 30. Then you do 4 – 3 = 1. Finally, you add 31 to give 31 as a result. For 43 – 18, you first do 40 – 10 = 30, 13 – 8 = 5, 30 – 10 = 20, 20 + 5 = 25. To use the partial decomposition strategy, you first subtract the first number and the tens of the second number. Then you subtract this result and the units of the second number. For example, for 54 – 23, you first do 54 – 20 = 34. Then you do 34 – 3 = 31 and give 31 as a result. For 43 – 18, you first do 43 – 10 = 33, 33 – 8 = 25.

The second difference with Experiment 1 was that half of the problems were presented horizontally and half were presented vertically. The horizontal presentation was the same as in Experiment 1. In the vertical presentation, decade and unit digits were displayed on top of each other in the center of the computer screen. The order of presentation was random. In both presentation modalities, each digit was presented in a predefined but not apparent 6 × 3.5-cm grid, subtending 5.0° (height) and 3.3° (width) visual angles.

Results
As in Experiment 1, results are presented in two main parts, one each for strategy use and strategy execution.
Age-related differences in strategy use

Variability in strategy use. No participants used full decomposition on either 0 or 100% of problems, and no problems were solved with full decomposition by either all or none of the participants (see distributions of strategy use in Table 5).

We analyzed mean percentages of use of full decomposition with an ANOVA involving a 3 (Group: adults, seventh graders, or fifth graders) × 2 (Presentation Modality: horizontal or vertical) × 2 (Borrow: present or absent) design with age as the only between-participant factor.

As can be seen from Fig. 2, participants of all age groups used both strategies approximately equally often (mean percentage use of full decomposition = 45%), main effect of age, F < 1. Participants used the full decomposition strategy more often when they solved vertically presented problems than when they solved horizontally presented problems (49 vs. 41%), F(1, 53) = 8.40, MSE = 370.68. Also, they used full decomposition more often to solve no-borrow problems than to solve borrow problems (57 vs. 33%), F(2, 54) = 13.84, MSE = 2069.63. No other effects were significant.

Determiners of strategy use. As in Experiment 1, we ran correlation and regression analyses to determine whether participants adjusted their strategy use to problem and/or strategy characteristics. Problem-based correlations in each age group and for each presentation modality were calculated between mean percentage use of full decomposition on each problem and two types of variables, namely, problem and strategy characteristics. Strategy performance variables were the same as in Experiment 1 and were calculated with the same formula (i.e., relative strategy speed and relative strategy accuracy). Problem characteristics involved size of correct differences (e.g., 14 for 31 – 17), borrow problems (coded 1) versus no-borrow problems (coded 0), and size of first operand (31) and second operand (17). The correlation matrix is presented in Table 6, collapsed over presentation modality because the same patterns of correlations were observed in both modality presentations.

The influence of problem and strategy characteristics on strategy use was not the same in each age group. Adults and seventh graders were much more likely to use the full decomposition strategy on borrow problems than on no-borrow problems. They were also much more likely to use it when it was faster than the partial decomposition strategy. Fifth graders were more likely to use the full decomposition strategy on borrow problems than on no-borrow problems. These results were confirmed in stepwise regression analyses conducted separately in each age group. In adults, the best predictor of full decomposition was borrow (R² = .73), followed by relative strategy speed, which accounted for 4% additional variance. Seventh graders’ use of full decomposition was predicted first by borrow (R² = .36) and then by relative strategy speed (partial R² = .10). The only single predictor of fifth graders’ use of full decomposition was borrow (R² = .36).

Age-related differences in strategy execution

Effects of age, presentation format, and borrow on performance. Mean solution latencies and percentage errors were analyzed with ANOVAs. ANOVAs were run with 3 (Group: adults, seventh graders, or fifth graders) × 2 (Strategy: full decomposition or partial decomposition) × 2 (Borrow: present or absent) × 2 (Presentation Modality: horizontal or vertical) designs with repeated measures on the last three factors (see means in Table 7). Again, only data from the no-choice condition are presented because data from the choice condition showed the same patterns of effects.

As can be seen from Table 7, adults were faster (7.1 s) than seventh graders (13.6 s), who in turn were faster than fifth graders (16.9 s), F(2, 53) = 9.53, MSE = 397.02. Moreover, participants were faster while solving no-borrow problems (10.1 s) than while solving borrow problems (14.9 s), F(1, 53) = 120.39, MSE = 22.00. This borrow effect varied with age groups, F(1, 53) = 7.28, MSE = 22.00, such that the borrow–no-borrow differences were 7.2 s in fifth graders, F(1, 17) = 64.13, MSE = 6.08, 4.4 s in seventh graders, F(1, 17) = 43.20, MSE = 15.63, and 3.1 s in adults, F(1, 19) = 64.13, MSE = 6.08. Finally, the Strategy × Presentation Modality × Borrow interaction was significant, F(1, 53) = 7.61, MSE = 13.86. It showed that borrow effects were comparable when they used the partial-decomposition strategy (6.0 s) than when they used the full-decomposition strategy (4.6 s) under the vertical presentation condition but were larger when participants used the full-decomposition strategy (5.6 s) or partial-decomposition strategy (3.2 s) under the horizontal presentation condition.
Analyses of errors showed the following significant effects: age, $F(1, 53) = 4.17$, $MSe = 107.65$; borrow, $F(1, 53) = 57.06$, $MSe = 78.52$; Age × Borrow, $F(2, 53) = 5.52$, $MSe = 78.52$; Strategy × Borrow, $F(1, 53) = 4.95$, $MSe = 14.37$; Age × Strategy × Borrow, $F(2, 53) = 6.07$, $MSe = 14.37$; and Strategy × Presentation Modality × Borrow, $F(1, 53) = 6.25$, $MSe = 5.21$. These effects showed that (a) adults (2.7%) were more accurate than either seventh graders (5.3%) or fifth graders (5.9%); (b) borrow effects were larger in children (although significant in all age groups, $F$s > 16.48, borrow–no-borrow differences were 8.1% in fifth graders and 8.3% in seventh graders) than in adults (2.5%); and (c) children made more errors on borrow problems than on no-borrow problems when they used either the full decomposition strategy or the partial decomposition strategy, $F$s > 3.56, whereas adults made errors equally often on borrow and no-borrow problems when they used either strategy, $F$s < 1. Borrow effects tended to be larger when seventh graders used the full decomposition strategy compared with the partial decomposition strategy and were of equal magnitude when fifth graders used either strategy. Finally, borrow effects were larger with horizontally presented problems (7.8%), $F(1, 53) = 56.85$, $MSe = 31.35$, than with vertically presented problems (6.3%), $F(1, 53) = 36.55$, $MSe = 31.00$, when participants used the

![Fig. 2. Mean percentage use of full decomposition strategy for carry and no-carry problems in each group of participants and each presentation modality (Experiment 2).](image-url)
full decomposition strategy. When participants used the partial decomposition strategy, borrow effects were larger while solving vertically presented problems (5.9%), $F(1, 53) = 46.22$, $MSe = 22.88$, than while solving horizontally presented problems (5.2%), $F(1, 53) = 38.38$, $MSe = 20.36$. No other effects were significant on participants’ performance.

**Age-related changes in the speed of borrow processing.** To analyze how the speed of borrow processing changes with age, we conducted componential analyses similar to those conducted in Experiment 1 (see Table 4, lower part, for mean component scores across group, presentation format, and strategy).

The same outcomes were observed for vertically and horizontally presented problems. Effects of age on intercepts for vertically presented problems, $F(2, 53) = 6.59$, $MSe = 95.01$, and horizontally presented problems, $F(2, 53) = 8.25$, $MSe = 75.47$, were significant, as were effects of age on slopes, $F(2, 53) = 4.35$, $MSe = 34.37$, and $F(2, 53) = 11.83$, $MSe = 20.70$, respectively. For both slopes and intercepts, there were no significant effects of strategy, $F_s < 1.82$, or Age $\times$ Strategy interaction, $F < 1$. Thus, as in addition (Experiment 1), as children grow older, they become more proficient at processing carries and at other arithmetic processes (number encoding, answer production, and columnar retrieval) while they use the partial decomposition strategy or the full decomposition strategy to solve problems presented either vertically or horizontally.

**General discussion**

The current experiments documented children’s age-related differences in complex arithmetic strategy use and execution. They replicated previous findings regarding children’s complex arithmetic skills and documented age-related changes in strategic aspects unknown before. Two sets of interesting findings, one each concerning strategy use and execution, have implications for further

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$p < .05$.

$p < .01$. 
understanding age-related changes in strategic behaviors, in general, and in complex arithmetic, in particular. Here we discuss implications concerning strategy use and strategy execution.

Strategy use in complex arithmetic

The first interesting set of findings concerns strategy use. The current experiments showed that, like in simple arithmetic (e.g., Barrouillet, Mignon, & Thevenot, 2008), both adults and children did not use a single strategy to solve two-digit addition and subtraction problems. Strategy use was influenced by problem, strategy, and participant characteristics in Experiment 1 and by problem and presentation format in Experiment 2. This is interesting because the only previous study that contrasted the relative influences of strategy and problem characteristics on children’s strategy use was Lemaire and Lecacheur’s (2002b) study on computational estimation. The data showed that the relative influence of these two variables changed with children’s age. Thus, it is possible that the relative influences of problem and strategy characteristics on strategy use might vary with both participants’ age and specific domains or tasks.

In Experiment 2, both children and adults used the partial decomposition strategy more often when problems were presented horizontally than when problems were presented vertically, and both children and adults used the two strategies equally often while solving problems presented vertically. This may have arisen because the partial decomposition strategy is easier to execute than the full decomposition strategy when problems are presented horizontally. Such contextual effects on how participants solve complex arithmetic problems are consistent with comparable previous effects in simple arithmetic. For example, previous works in simple arithmetic showed that participants’ performance differed with varying number formats (e.g., LeFevre et al., 2001; Noël et al., 1997), different levels of speed–accuracy pressure (Campbell & Austin, 2002; Lemaire, Arnaud, & Lecacheur, 2004), and whether verbal protocols were collected during problem solving (Kirk & Ashcraft, 2001). Future empirical studies should replicate and further investigate effects of presentation format to understand such contextual effects and to help further flesh out how such task characteristics influence how children change their approach to solve complex arithmetic problems.

Regarding determiners of strategy choices, there were interesting differences between addition and subtraction problems and as a function of participants’ age. To solve addition problems, adults chose strategies on each problem on the basis of first strategy characteristics (relative strategy precision) and then problem features (presence/absence of carry). Children chose strategies on the basis of strategy characteristics only (relative strategy precision in fifth graders and relative strategy speed in third graders). To solve subtraction problems, both adults and seventh graders based their strategy choices first on problem feature (presence/absence of borrow) and then on relative strategy speed. Fifth graders’ strategy use was predicted by the presence or absence of borrow only.

### Table 7
Mean solution times and percentage errors for full and partial decomposition strategies in each group of participants and for each problem type and each presentation condition (Experiment 2, no-choice conditions)

<table>
<thead>
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<th>Solution times</th>
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<td>No-borrow</td>
<td>Borrow</td>
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<tr>
<td>Fifth graders</td>
<td>12.6</td>
<td>19.1</td>
</tr>
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*Note.* Solution times are in seconds (s).
Moreover, there were no age differences in the amount of variance predicted (as revealed by comparable \( R^2 \) values in analyses predicting strategy use) while solving addition problems. Such differences were observed in solving subtraction problems. Indeed, \( R^2 \) values were .77, .46, and .36 in adults, seventh graders, and fifth graders, respectively. Such differences between operations may stem from younger children basing their subtraction strategy choices on other variables not assessed here. It may also come from their choosing strategies on the same bases, but as children become more expert at solving subtraction problems, whether a problem is a borrow problem is influencing their strategy choices more and more strongly.

Observing that calibration of strategy choices to problem and strategy characteristics differed as a function of arithmetic operation and participants’ age was interesting from a theoretical perspective. Indeed, although computational models of strategy choices, such as SCADS (Siegler & Araya, 2005), have been built to account for simple arithmetic, they can be generalized to complex arithmetic. SCADS assumes that correlations between strategy choices and strategy performance are the result of increased expertise of participants. This increased expertise is correlated with participants’ greater knowledge of relative costs and benefits of strategies. This knowledge of relative strategy performance is stored in a database that is activated while participants solve problems and choose among strategies. The strategy that works best for a given problem is selected. This predicts effects of strategy performance on strategy choices. In the current Experiment 2, there were no effects of strategy performance on fifth graders’ subtraction strategy choices, in contrast to such effects in adults and seventh graders. In addition problem solving (Experiment 1), strategy choices were influenced by strategy performance in all age groups. Following SCADS, fifth graders did not take the relative efficiency of strategies into account while choosing among subtraction strategies, possibly because their database did not have enough information regarding relative strategy performance. It is possible that fifth graders did not have enough experience with the partial decomposition and full decomposition subtraction strategies. This predicts that with enough experience, fifth graders’ strategy use might be influenced by relative strategy performance. Such a prediction could be tested in a microgenetic study. Fifth graders could be asked to solve subtraction problems with the full and partial decomposition strategies during several sessions. Observing how fifth graders’ strategy choices become more and more influenced by relative strategy performance would support SCADS prediction.

Strategy execution in complex arithmetic

The second set of interesting findings in this study concerns strategy execution. Contrary to previous works on complex arithmetic, participants’ strategy execution was not biased here by selective use of strategies across problems and/or participants. Performance improved greatly with age, as expected, on both addition and subtraction problems. Most interesting were the carry and borrow effects and the \( \text{Age} \times \text{Carry/Borrow} \) interactions, showing that carry and borrow effects (better performance on no-carry problems than on carry problems and better performance on borrow problems than on no-borrow problems) decreased in magnitude as participants’ age increased. Componential analyses revealed that increased performance with age stemmed from increased efficiency of processing carries and borrow as well as of other processes (e.g., encoding, columnar retrieval, responding).

Also, performance varied with strategies. When they solved addition problems, participants made more errors with the partial decomposition strategy than with the full decomposition strategy, although they were equally fast with both strategies. Although this may be a specific instance of similar solution times resulting from different cognitive processes, this was surprising because both strategies differed in the number of processes as well as in the type of processes. The partial decomposition strategy involves two steps (e.g., to solve 43 + 54, participants first do 43 + 50 = 93 and then 93 + 4 = 97), whereas the full decomposition strategy involves three steps (e.g., participants first do 40 + 50 = 90, then 3 + 4 = 7, and finally 90 + 7 = 97). It is possible that the difficulty of each step is more crucial than the number of steps, such that the two steps of the partial decomposition strategy are harder to execute than the three steps of the full decomposition strategy. Assessments of duration of each step would enable this explanation to be tested.

To conclude, the current experiments revealed important effects of strategies, age, problem types, and situational constraints on how participants solve complex arithmetic problems and on their
performance. These findings reveal that, above and beyond specificities of complex arithmetic (e.g., specific strategies, monitoring partial results, processing carries), complex and simple arithmetic have a lot in common from a cognitive perspective. Variability in selection of strategies is common across simple and complex arithmetic. Accordingly, it is crucial to assess participants’ performance as well as age-related changes in strategy selection and strategy execution. Future research taking a strategy perspective will provide deeper understanding of age-related changes in children’s complex arithmetic skill.

Acknowledgment

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References


